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THE LOCALIZATION OF OBJECTS IN THE HUMAN BODY

by A. BOUWERS. 621.386.16 : 616.073

X-ray photographs of objects in the human body permit the localization of an object in two dimensions. In order to determine the third coordinate, the depth, stereoscopic X-ray photographs, so-called planigraphic X-ray photographs and finally an apparatus by which the object is localized as the point of intersection of two beams of X-rays may be used. A description is given of such an apparatus, which has recently been developed in the Philips factories, and which is intended especially for the finding of projectiles in the body.

Introduction

One of the tasks of medical diagnostics is the accurate determination of the position of objects in the human body. As examples we may mention the localization of tumors in the stomach or other organs, or of cavities in the lungs due to a tubercular process. In wartime the problem takes on a particular significance due to the fact that the surgeon is often concerned with wounds where projectiles have remained in the body. In order to remove such projectiles with the least harm to the patient the first essential is an accurate determination of their position.

Ever since the discovery of X-rays, the X-ray shadow photograph has been used as a guide in such cases. In the first instance, however, this gives only a localization of the object in two dimensions. Methods have long been sought of obtaining information from the X-ray photographs about the position in the third dimension also (the depth under the skin). When the possibilities are reviewed which present themselves for this purpose, it seems obvious to start from the phenomena which make it possible for the eye to obtain a plastic impression in normal vision. There are in the main three such phenomena: the partial screening of distant objects by those lying closer, the decrease with increasing distance of the angle of vision within which an object of a given size is seen, and finally the parallax occurring due to the collaboration of the two eyes.

The first possibility mentioned — plastic impression due to partial screening — is missing in the X-ray photograph, since only shadows are seen, and in the first instance the shadow picture obtained of two objects lying one behind the other is independent of the order in which they are traversed by the X-rays.

The second possibility also — perspective diminution — cannot offer much aid in the X-ray picture. Since the thickness of the body is generally much smaller than the distance to the source of X-rays, the angle within which a given object is seen from the source can only vary slightly with its position, and, moreover, in order to translate the slight differences in size of the shadows in the X-ray picture into differences in distance, it would be

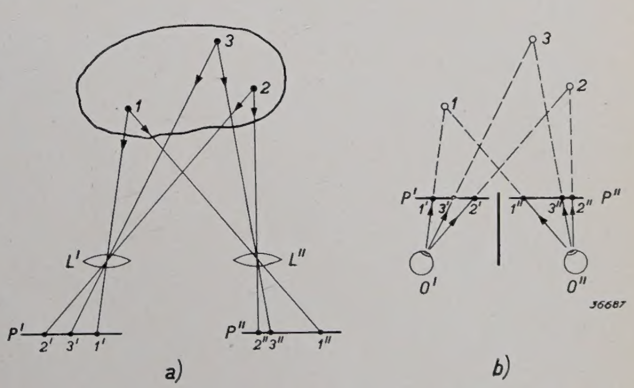


Fig. 1. When a three-dimensional object (points *1*, *2*, *3* in *a*) is seen, two different images are formed on the retina *P'*, *P''* of the two eyes (*L'*, *L''* eye lens). A plastic impression is obtained upon the flowing together of these two images into one. Stereoscopic photography is also carried out in the manner indicated in *a*), where *P'*, *P''* are two photographic plates and *L'*, *L''* two photographic lenses. When the two plates are observed in the manner indicated in *b*) (*O'*, *O''* eyes) the impression of a three-dimensional image is again obtained.

necessary to know accurately the size of the objects themselves. This is generally not the case.

The third possibility remains — the parallax. In *fig. 1* it is shown how normal stereoscopic vision or observation of stereoscopic photographs takes place with the help of the parallax. The essence of the method consists in the fact that *each point on the object is localized as the point of intersection of two rays of light.* This principle can also be applied in X-ray diagnostics and forms indeed the basis of all the usual methods of determination of depth in X-ray pictures. We shall here review several of these methods and describe in more detail an apparatus for finding projectiles which has been developed in the Philips laboratory.

Stereoscopic X-ray photographs

The diagram of fig. 1 cannot serve directly for X-ray pictures, since in taking such pictures the points of the object cannot emit any radiation but only intercept it. *Fig. 2* shows the principle of the photography and observation of X-ray stereoscopic pictures.

It is obvious that in order to superpose the two different pictures to give a (plastic) whole, the observer must be able to see every point of the one on the other. In X-ray pictures this is not so easy as in the observation of ordinary photographs. The sharp contours are often missing which could immediately be identified, and in their place there are more or less vague shadows whose lack of definition may sometimes be of the same order of magnitude as the shift due to parallax! It is therefore very difficult or even impossible to obtain a plastic impression of many objects, such as diseased spots in the lungs, by the observation of stereoscopic X-ray photographs.

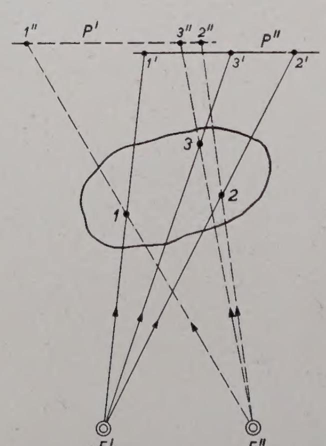


Fig. 2. In taking stereoscopic X-ray photographs shadow images are thrown by two foci F', F'', at a distance apart of 5 cm, successively upon two correspondingly placed plates P', P'' and photographed. The observation of the photograph may in principle take place in the same way as if the eyes occupied the same position as the X-ray foci.

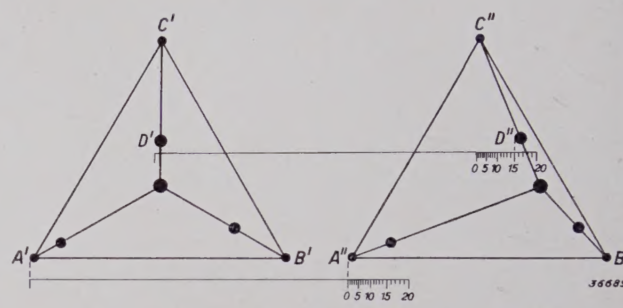


Fig. 3. In the application of a standardized method in which the two X-ray stereophotographs are taken side by side on a single film, the depth of each point can be measured directly with a calibrated ruler. In the case here represented a tetrahedron with small spheres at certain points on its edges is photographed. It may be read off directly that point D lies 15 cm above the plane of the points ABC.

The stereoscopic X-ray pictures taken according to the scheme of fig. 2 can also be used for an *objective* determination of the "depth" of the different points, without trying to see a plastic picture. For this purpose the relative displacement of corresponding points on the two pictures must be measured and calculated, a procedure which is very much facilitated when the geometrical relations prevailing during the making of the exposures are permanently fixed, and a ruler is used to measure the displacement, which is calibrated directly in depths (with respect to a given plane of comparison) [1]), see *fig. 3*. When this method is used the visualization of the object is of course entirely lost; the doctor must reconstruct the situation of the object in the body from a series of values which he has obtained in this way.

Planigraphy

An interesting method of obtaining an objective determination of depth and at the same time a physical visualization of it is by means of planigraphy [2]) (also called tomography). Suppose that the projections of the object from the two foci which were used in taking the stereoscopic X-ray pictures are not photographed on two different plates, but successively on the same plate (see *fig. 4*). By moving the plate after the first exposure

[1]) This is described by W. Hondius Boldingh, Vereinfachte und standardisierte Stereotechnik, Röntgenpraxis **1**, 561, 1929.

[2]) This method was published independently and almost simultaneously by D. L. Bartelink (Fortschr. Röntgenstr. **47**, 399, 1933), B. C. Ziedses des Plantes (Fortschr. Röntgenstr. **47**, 407, 1933) and A. Vallebona (Fortschr. Röntgenstr. **48**, 599, 1933). However, A. E. M. Bocage had already obtained a patent on it in 1921 (French patent No. 536.464).

from position *1* to position *2*, the shadow of point A can be made to fall on the same point in both exposures $A' = A''$. Point A is thus sharply focussed, and this is still true when instead of making two

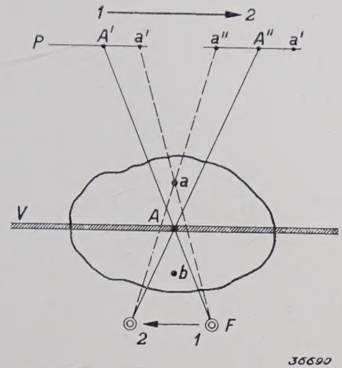


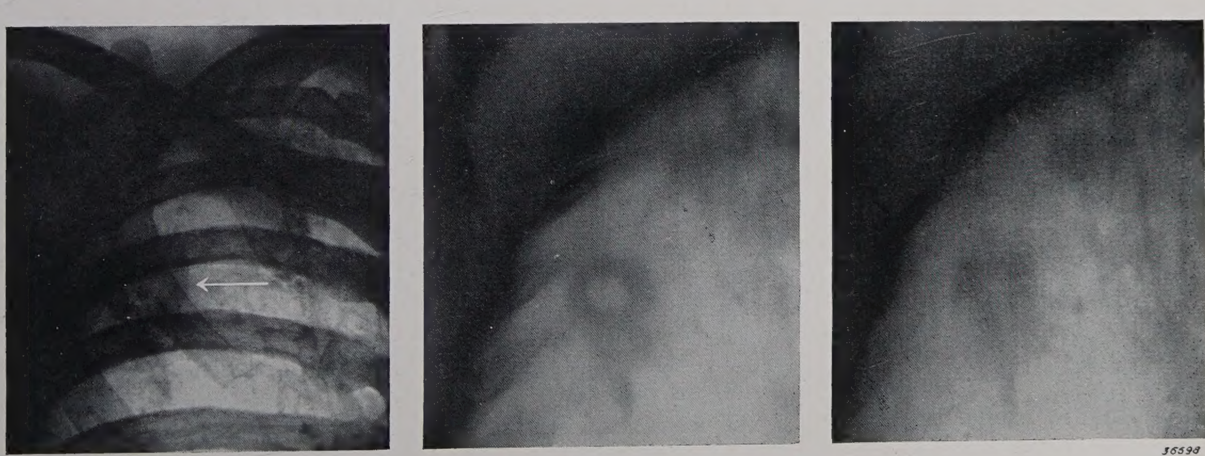
Fig. 4. Principle of planigraphy. During the exposure the X-ray focus F is moved from *1* tot *2*, and at the same time the photographic plate P is moved correspondingly. All points in the plane V are sharply focussed. while the shadows of points outside this plane (a, b) are rendered undefined.

separate exposures plate and focus are shifted continuously during the exposure from position *1* to position *2*. If we now consider point a, we see that its shadow on the plate is displaced from a' to a''. The shadow of point a thus loses all definition. Further consideration shows that in the exposure described with moving plate and focus, not only point A but all points on plane V are sharply focussed, while the shadow of points above this plane (a) and below it (b) are spread out along a line of a certain length, and therefore lose all definition. An X-ray picture is thus obtained in which all objects in the cross section V of the irradiated body are visible as sharply defined shadows, while the rest of the body causes a more or less uniform fog over the picture. In practice focus and plate are usually not made to move along a straight line, but over a longer traject which as nearly as possible fills a whole area (for instance a spiral or a narrow sine curve) and over which traject focus and plate always maintain proportional velocity in opposite direction. The blotting out of the undesired planes is thereby considerably improved. If, for instance, it is desired to find the position of a cavity in the lungs by this method, a number of such planigraphic exposures can be made in each of which a different cross section (of known depth) through the body is brought into focus; afterwards the exposure can be picked out upon which the cavity is sharply (or most sharply) defined. In *fig. 5* an example is given of such exposures of different cross sections, together with an ordinary X-ray photograph of the same patient [3]).

The Philips "bullet finder"

For the finding of projectiles mentioned in the introduction the methods described are less suitable. In such a case it is particularly important to be able to determine the position quickly and

[3]) It must be noted that only under certain circumstances conclusions may be drawn from the planigraphic image obtained about the shape of the objects. Various investigations have been carried out in recent years on this subject, among others by the Medical Department of the Philips concern, see for instance G. C. E. Burger and J. G. A. van Weel, Fortschr. Röntgenstr. **57**, 143, 1938; further R. H. de Waard, Ned. T. Geneesk. **83**, 368, 1939.



a b c

Fig. 5. *a*) Ordinary X-ray photograph of part of the chest of a patient. A ring-shaped shadow (indicated by the arrow) is visible, which indicates the presence of a cavity in the lung.
b) "Planigram" of the same patient. The photograph shows a cross section 7 cm below the skin of the back. The cavity is very clear and sharply defined.
c) Planigram at a depth of $8^1/_2$ cm. The cavity in this case has lost almost all definition.

in a simple way, and to obtain the result in such a form that it can serve the surgeon immediatly as a guide during the operation. It will be clear that the methods described would be much too elaborate in such cases, when it is kept in mind that a shift in position of the object to be removed may often occur during the operation. It is then impossible to wait for the required new determination of position by the exposure and development of one or more photographs.

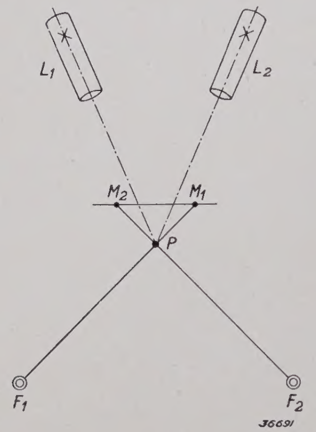


Fig. 6. Principle of the "bullet finder". Four beams meet at point P: two beams of light (broken lines) emitted by the lamps L_1, L_2, and two beams of X-rays along the lines joining two foci F_1, F_2, and two marks M_1, M_2 on a fluoroscope screen.

The numerous methods which have been worked out to satisfy the requirements of this particular application [4]) are all based upon the principle already mentioned: the localization of a point as the point of intersection of two beams. The "bullet finder" which has recently been developed at Philips is also based upon this principle. In this apparatus four ray directions intersect each other at a definite point P (see *fig. 6*): two light rays L_1 and L_2 and two rays R_1 and R_2 which are fixed as the lines joining two X-ray foci F_1 and F_2 and two marks M_1 and M_2 on a fluorescent screen. If the body of the patient is placed in the path of the rays so that the shadow of the projectile which F_2 casts falls on M_2, and that from F_1 on M_2, the projectile is obviously situated exactly at point P which is distinguishable to the surgeon as the point of intersection of the two light rays [5]).

Since the position of the patient is fixed by other factors concerned with the operation, the body with the projectile is not brought to point P, but point P to the projectile. Furthermore, two X-ray tubes are not used simultaneously, but only one tube whose focus is brought successively into the positions F_1 and F_2. This is desirable in order not to administer to the patient an unnecessarily large dose of X-rays, and in order not to obtain two superposed images on the fluorescent screen, which would be very disadvantageous as concerns the image contrast, and if several projectiles are present it would also make the images confusing. In *fig. 7* the apparatus and its method of functioning is shown diagrammatically. The standard which supports the X-ray tube, the fluoroscope screen and the light sources on a sliding frame is pushed close to the operating table in such a way that the X-ray tube, first set in position *1*, is beneath the patient, and the shadow of the projectile falls upon the mark M, of the fluoroscope screen D. This is checked by an assistant who can easily observe the fluorescent image in the mirror S by looking into the tube K, without it being necessary to darken the room. After this adjustment, by which the focus is placed directly under the projectile, the assistant fixes the standard by means of a foot brake with

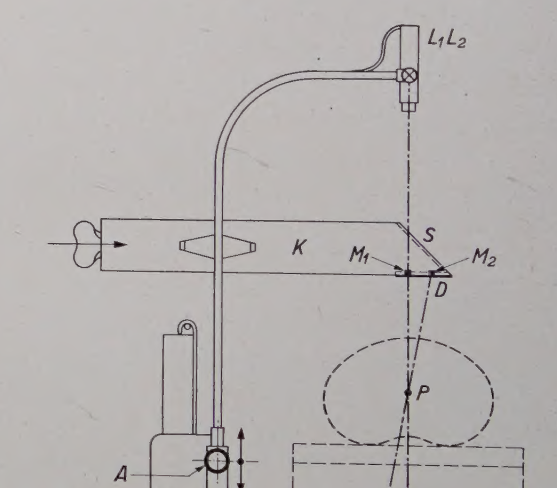


Fig. 7. Construction of the "bullet finder". *1*, *2* positions into which the X-ray tube R can be brought. M_1, M_2 marks on fluoroscope screen D, S mirror, K tube; the arrow indicates the direction of observation of the operator of the apparatus. L_1, L_2 two projection lamps lying exactly one behind the other in this sketch. The point P is made to coincide with the projectile. A handle for manipulation, V foot brake.

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[4]) A survey is given in the book of R. Grashey, Steckschuß und Röntgenstrahlen, G. Thieme, Leipzig 1940.

[5]) Light beams have already been used for this purpose, although in a manner different from that here described, by C. Chaussée; see J. belge Radiol. **23**, 305. 1939.

respect to horizontal displacement, brings the X-ray tube into position *II* (see fig. 7) and then shifts the movable frame in a vertical direction until the shadow of the projectile falls upon mark M_2. The tube with the fluorescope screen is then pushed back and the operation can be begun. The two light spots which are thrown upon the patient's skin by the projection positions L_1 L_2 (these can be seen better in the photographs *fig. 8* and *9*) serve to guide the surgeon. The directions of the two light beams make an angle of about 20° with each other, so that the depth of the point *P*, *i.e.* of the projectile, under the skin of the patient is about 3 times the distance between the two light spots.

It is of essential importance for a rapid localization that the first adjustment should not be destroyed later by the second adjustment. This is guaranteed by the fact that in the second adjustment the imaginary line I-M_1 coupled to the movable frame is displaced vertically, *i.e.* in its own extension, and therefore always continues to pass through the point *P*. Experience has shown that in this way the position of the object can be determined within a few millimeters in one minute. During the operation also the two beams of light continue to indicate the direction. If necessary the observation can be repeated without appreciable interruption of the operation in order to find out whether the object has moved. In fig. 8 a photograph is given of the whole set up [6]).

The fluoroscope screen must lie at least as far above the point *P* as the thickness of a patient may be. Due to this relatively large distance the geometric lack of sharpness of the X-ray picture (half shadow width, caused by the finite dimensions of the focus), is about $^3/_4$ of the width of the focus. In order to make a sufficiently sharp localization pos-

[6]) The apparatus described has already given good service to hundreds of wounded, see J. Schlaaff, Zentralbl. Chirurgie **67**, 1924, 1940.

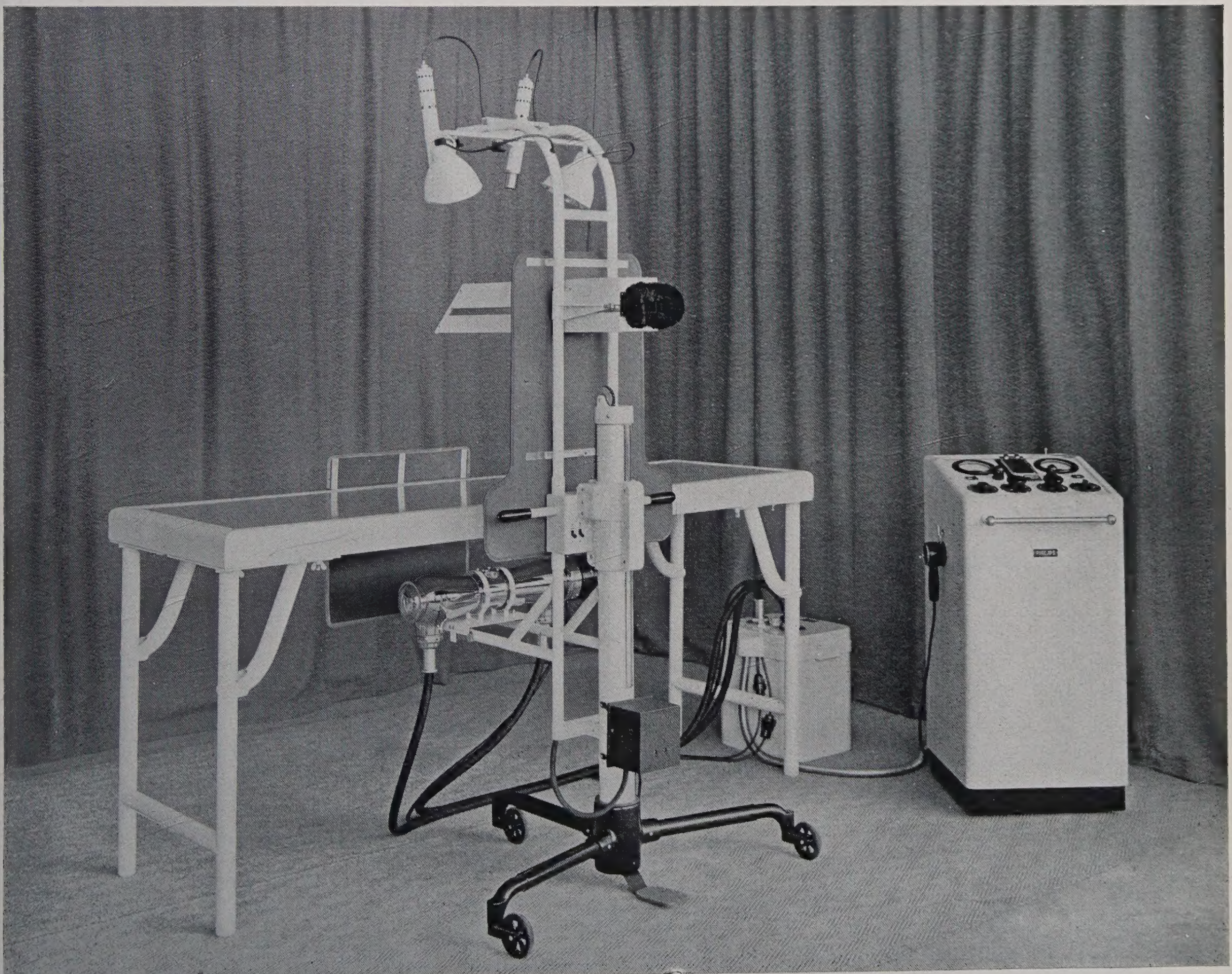

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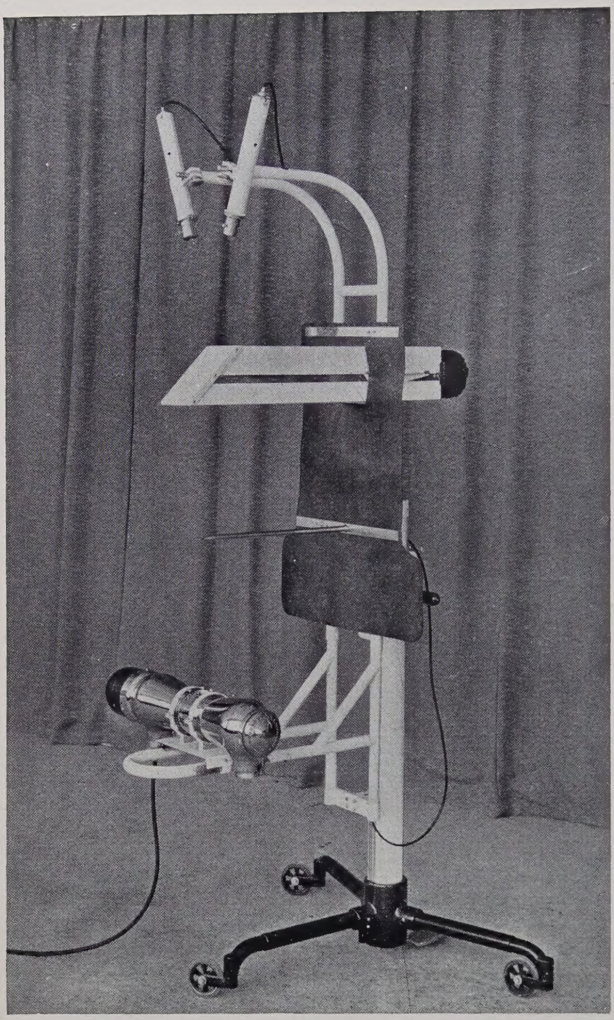
Fig. 8. Complete arrangement for the localization of projectiles. In the middle the bullet finder beside the operating table. On the right the desk from which the X-ray tube is operated.



sible, therefore, an X-ray tube with a very fine focus (apparent dimensions 1.7 × 1.7 mm) is used.

When the adjustment of the apparatus, *i.e.* the intersection at point P of all four beams, is destroyed by slight deformations of the standard, such as might occur due to transportation, it can be adjusted anew with the help of the iron rod visible in fig. 9. The point of the rod is placed at the intersection of the two X-ray beams, $1\text{-}M_1$ and $2\text{-}M_2$, and the two projectors L_1 and L_2 are then turned so that the two light beams again intersect each other on the point of the rod.

Fig. 9. Photograph of the "bullet finder". The two projection lamps may be seen at the top, below them the tube with the fluoroscope screen, which can be pushed back for the operation. The rod between the tube and the X-ray tube is only inserted for the adjustment of the whole apparatus.

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WIRELESS TELEPHONY WITH MOVING MOTOR-CARS

by C. G. A. von LINDERN.

621.396.5.029.6 : 621.396.931

In order to obtain an easy method of communication between the central station and motor-cars of various public services (fire department, police, etc.) the cars may be provided with a small ultra-short wave transmitter and receiver. The transmitting-receiving installation DR 38, which works on a wave length of 4 to 4.5 m has been constructed for this purpose. A description is here given of the transmitter and of the receiver, which contains a super-regenerative detector. The method of functioning of this detector is explained. In conclusion the supply, the aerial and other details of the installation are discussed. In experiments in which the aerial of the central station was set up at a height of 45 m, it was found possible to obtain a good connection over distances of from 10 to 20 km.

It has become more and more customary to equip motor-cars which are used by the police, the fire department, large garages and other services, with radio transmitters and receivers which make a telephonic connection possible between the motor-car and the central station. The great practical importance of such a possibility of communication for the services in question is obvious: the central station can by this means give orders to the cars out on patrol duty or en route to render assistance, and conversely the cars can report, ask instructions or reinforcements if necessary, etc.

In the case of other means of conveyance such as ships and aeroplanes, the use of transmitter-receiver installations has long been familiar. The requirements made of such an apparatus for cars are, however, considerably heavier in certain respects than for ship or aeroplane installations. Only a very limited amount of space is available either for the apparatus or the aerial; the energy consumption is limited to that which the accumulator battery of the car can provide; furthermore in the case of a moving car in a city the very high level of interference must be taken into account. Finally the installation must of course be able to be operated by a layman, since it would be impossible to have a radio-telegraphist in every car.

The consequence of these and other requirements for the construction of such an installation will be discussed here, and the construction of the Philips car installation, type DR 38, which has been developed on the initiative of the "Committee for special radio services at very high frequencies" will be described in particular.

Choice of the wave length

In order to obtain satisfactory radiation with a limited energy consumption, an aerial with a high radiation efficiency must be used. This amounts practically to the fact that the length of the aerial must be $^1/_4$ or $^1/_2$ of the wave length. The choice of the wave length is hereby immediately limited to the region below 10 m, because it is obvious that it is impossible to mount a structure many metres high as aerial on a car. The high level of interference in the cities also leads to a similar limitation in the choice of the wave length: with the small power available the car transmitter has no chance of dominating over the prevailing interferences; it must therefore try to avoid the frequency spectrum of those interferences as far as possible. At very high frequencies, above about 60×10^6 c/s, the interference level is sufficiently low. The installation DR 38 works on wave lengths from 4 to 4.5 m ($75 - 67 \times 10^6$ c/s). These very short waves already experience an appreciable shadow effect due to obstacles in their path such as hills, woods, groups of buildings, etc., so that for a good connection between the car and the central station it is desirable that there should be no obstacles in a straight line between the two aerials. This, however, offers no difficulty since the aerial on the central station can, without much trouble, be mounted at such a great height that it is "visible" 20 or 30 km away, and for the services mentioned this is sufficient, since the cars need in general only to travel within a relatively limited district.

The transmitter

As oscillator the transmitter contains a back-coupled triode (TE 05/10). Tuning and maintaining a constant frequency of the oscillator at such short waves is often accomplished with the help of a Lecher system (a socalled long line); because of the limited amount of space, however, this method was not used in our case. An LC circuit was therefore used, which corresponds in principle to the circuits for broadcasting wave lengths, but which is constructed quite differently because of the losses occurring at high frequencies. In *fig. 1* the construction is shown and explained. The ohmic, dielectric and radiation losses are here limited to a minimum,

so that a very sharp resonance curve, and thus a high frequency stability, is obtained. By screwing the cover D more or less tight the capacity can be varied and the resonance frequency thus regulated

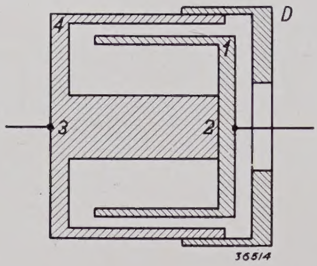


Fig. 1. Construction of the low-loss oscillation circuit which fixes the frequency of the transmitter. The construction can be considered to be formed by a rectangular current loop *1234*, in which between *1* and *4* a capacity is connected, while the whole is allowed to rotate about the axis *23*. The magnetic lines of force which are responsible for the self-induction run in circles about the axis *23*, entirely within the solid of revolution obtained. The gap of the "loop" in which run the electrical lines of force which are responsible for the capacity, is so narrow that here also practically no lines of force reach the outside, so that the losses by induction in the surroundings and by radiation are only very small. Due to the large copper cross section of the current loop the ohmic losses are also slight, while by the use of air as a dielectric the dielectric losses are small enough to be neglected.

between 75 and 67 $\times$ 10^6 c/s; the regulation, however, need be carried out only after the wave length to be used, which must of course be different for different services in the same district, has been determined.

The circuit described is connected between grid and cathode of the transmitter valve, see the diagram of *fig. 2*. The frequency is therefore partially determined by the capacity between grid and cathode in parallel with the circuit capacity, which is apparently increased by the capacity C_{ag} between anode and grid by an amount $gC_{ag}R_a/(R_i + R_a)$, where g is the amplification factor and R_a and R_i the external and internal resistances, respectively, of the valve. Since R_a depends upon the aerial circuit, which is coupled with the anode circuit, the frequency will therefore be somewhat influenced by the aerial, and the previously regulated wave length may therefore change slightly upon mounting the apparatus in the car. When the car is moving the frequency will then still be able to vary slightly, due to the continually changing environment which affects the aerial circuit. In modulation, which is done by changing the anode voltage, the frequency will also vary, since changes hereby occur in the valve capacities, among others. In order to keep all these frequency variations small, the valve mentioned, TE 05/10, is so constructed that the capacities have only low values. (An acorn or button valve, which satisfies very high standards in this respect [1]) because of its small dimensions, could not be used here, since it cannot deliver enough power). At a modulation depth of 90 per cent the frequency now varies about 25 000 c/s, which is no longer disturbing because of the great band width which the receivers may have in this wave length region.

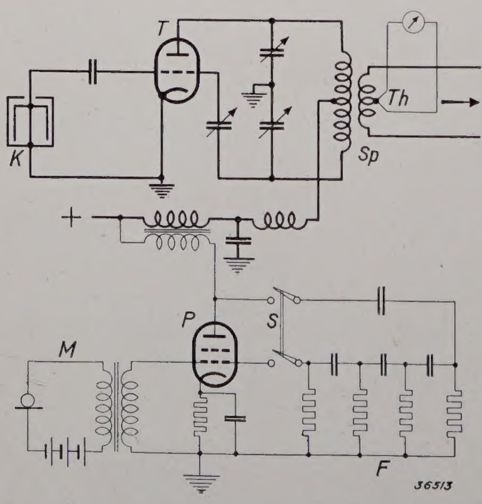


Fig. 2. Simplified diagram of the transmitter. T transmitter-triode, K low-loss circuit, *Sp* aerial coupling coil, *Th* thermo-element. The part of the connections drawn with thin lines serves for modulation. In this part P is a low-frequency pentode, M microphone circuit, F phase-rotating element for excitation of the call tone, S switch (signalling key) for calling (or telegraphy).

For modulation a low-frequency pentode is used whose grid A.C. voltage is supplied directly by the microphone. The modulator valve is used at the same time for the generation of the call signal which consists of a tone of 1 000 c/s. For this purpose part of the anode A.C. voltage of the valve is fed back to the grid *via* a filter (see fig. 2) which rotates the phase of voltages with different frequencies to different degrees, in such a way that the phase rotation only has the correct value to cause a building up of the vibration for a frequency of about 1 000 c/s. This feed-back can be set in operation with a switch in the form of a signalling key; in this way it is possible to give Morse signals as well as the call signal.

In the coupling coil with which the aerial is coupled with the anode circuit, a thermo element is included with which the current at this point can be measured. This permits a check on the functioning of the transmitter. The power delivered to the aerial by the transmitter amounts to 5 to 6 watts.

The receiver

The primary aim in the construction of the re-

1) See the photograph in Philips techn. Rev. **5**, 177, 1940.

ceiver was to have as few stages as possible, in order that it should occupy as little space as possible, and in order to limit the possibilities of defects as far as possible. On the other hand the receiver must in any case have a high sensitivity, while, due to the considerable variations in field strength which may occur while the car is moving, a good quick-acting automatic volume control is desired.

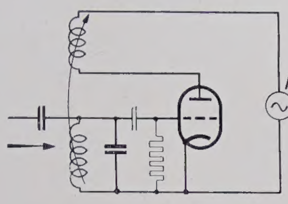


Fig. 3. Diagram showing the principle of a superregenerative detector. By means of the auxiliary oscillator H the anode voltage is so influenced that the circuit begins to oscillate periodically.

These requirements can be satisfied particularly well by the application of a so-called superregenerative detector. We shall examine this in somewhat more detail [2]).

Upon switching on an oscillator, for instance an amplifier valve with a back-coupled oscillation circuit (see *fig. 3*), the amplitude E of the oscillation generated at first increases according to an exponential law: $E = E_0 \exp(\alpha t)$, and then reaches a definite final amplitude due to the limitation by the valve characteristic. The time τ necessary for the building up of the oscillation to this final amplitude (we mean by this the amplitude at which the deviation from the exponential variation occurs) will depend upon the initial amplitude E_0. It is of importance in this connection that the difference $\tau_1 - \tau_2$ between the times for the building up with two different initial amplitudes E_1, E_2 is determined exclusively by the relation E_2/E_1, since

$$\tau_1 - \tau_2 = \frac{1}{\alpha} \ln E_2/E_1.$$

If we apply a signal to be received, for instance a modulated carrier wave, to the grid of the oscillator valve (the superregenerative detector), and if we change the anode voltage of the oscillator periodically in such a way that it can oscillate during a time T and the oscillation is then suddenly suppressed, the signal amplitude prevailing at the beginning of every period T functions each time as initial amplitude. During a period of time $\Delta = T - \tau$ in every period T the oscillator oscillates with maximum amplitude and large anode currents flow. In charging of the grid condenser by the grid current which increases simultaneously with the anode A.C., detection occurs in the ordinary way, *i.e.* the operating point on the characteristic of the valve continually shifts so that the anode D.C. varies in the same way as the envelope of the high frequency anode alternating current. In *fig. 4* this is shown diagrammatically. The time Δ during which the maximum anode direct current flows, therefore also the amount of charge flowing through the anode impedance per period T, varies from period to period with the signal amplitude. By a suitable choice of the anode impedance one obtains in this way relatively high low-frequency A.C. voltages whose variation with sufficiently small value of T forms a faithful picture of the variations of the signal amplitude received (modulation), and whose magnitude, according to the explanation given, is independent of the magnitude of the signal amplitude E and determined solely by the maximum ratio E_2/E_1, *i.e.* the depth of modulation of the signal. Thus on the one hand, with one valve one can obtain a very great amplification, and on the other hand an ideal "automatic volume control" is obtained. If there is no signal to be received then the spontaneous voltage fluctuations due to the thermal agitation of the electrons, etc. functions as initial signal for the periodic building up, and this is heard clearly as noise, just as is the case with a superheterodyne receiver with automatic volume control in the absence of a signal.

The periodic functioning of the oscillator can be accomplished with the help of a separate small

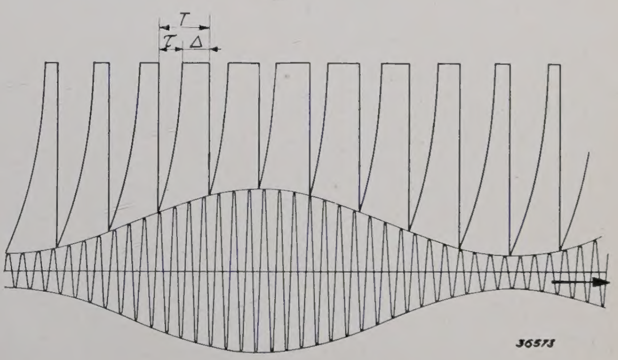


Fig. 4. The signal to be received (modulated carrier wave) and the variation of the rectified current through the valve. The high-frequency vibration is built up each time from the given signal amplitude, according to the envelope here drawn, to a constant final amplitude (time for building up α), and after a constant time T suppressed. For the sake of simplicity it is here assumed that after the exponential increase the amplitude suddenly becomes constant and upon suppression suddenly falls to the (new) initial value. (The oscillations are built up respectively much higher than according to this drawing).

[2]) *Cf.* E. H. Armstrong, Proc. Inst. Rad. Eng. **10**, 244, 1922; P. David, Onde él. **7**, 217, 1928; H. O. Roosenstein, Hochfrequenztechn. u. Elektroakust. **41**, 85, 1933.

oscillator valve, which, for instance, changes the anode voltage in the desired rhythm. As already pointed out, the period T must be chosen small enough to record a sufficient number of points on the modulation curve in the "scanning" (fig. 4). The auxiliairy frequency must therefore be considerably higher than the highest speech frequency to be reproduced. On the other hand, T may not be chosen indefinitely small. Above, in speaking of the times Δ of maximum generation, we assumed tacitly that T is always greater than the time for building up τ. For a sufficiently small initial amplitude, however, this will no longer be true, the oscillations then do not reach the final amplitude drawn in the figure at all. Signals below a certain amplitude limit, which is higher the smaller T is, are practically no longer reproduced. By variation of the auxiliairy frequency (T), therefore, the sensitivity of the receiver can be regulated, and at the same time, when it is possible to count on relatively strong signals, the noise in the intermissions of the reception can be decreased.

In the construction of the receiver in the installation here described, another valve was saved in the following way. The periodic generation of the oscillator is not accomplished here by an auxiliary oscillator, but takes place automatically, thanks to a suitable choice of dimensions for the condenser and the leakage resistance in the grid circuit and a sufficiently tight back-coupling. Upon oscillation the negative grid voltage rapidly becomes so large that the oscillations suddenly break off just after they have reached a certain value. The grid condenser is then discharged and the process begins anew (so-called "blocking") [3]). The action of the arrangement is now different is so far as the intervals of oscillation do not follow each other strictly periodically, but the oscillations, and thanks to the detection the anode D.C. also, vary in such a way that during each interval of oscillation about the same charge flows through the valve. The intervals succeed each other rapidly at high signal amplitude, at lower, less rapidly (see *fig. 5*). Further consideration shows that now also the variation of the average time value of the charges (*i.e.* the sound intensity obtained) is determined practically only by the depth of modulation of the signal received.

It must be pointed out that the automatic volume control, which can be applied in a superheterodyne receiver, must always work with a certain time lag in order not to regulate the modulation of the carrier wave out of existence; in superregenerative receivers on the other hand the regulation acts without any time lag, any interferences occurring (suddenly occurring signal with greater carrier wave amplitude) are therefore rendered harmless more satisfactorily.

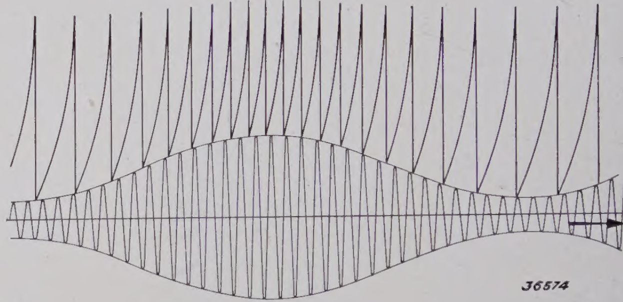


Fig. 5. Same as in fig. 4 upon making use of super-regeneration The building up now does not take place at constant intervals, but with large signal amplitude the intervals are smaller, with small signal amplitude, larger.

The oscillator valve used as superregenerative detector is an acorn valve (4 671). A stage of high-frequency amplification precedes the detector. This has certain advantages. A lower sensitivity of the detector is needed and the noises occurring in the intermissions of the reception are thereby decreased. The radiation of the receiving aerial caused by the periodic generation of the detector is considerably decreased so that other receivers in the neighbourhoud experience less interference. Furthermore, the high-frequency amplifier stage prevents the regular "blocking" of the detector in the correct manner and in the desired frequency region, which requires a very careful adjustment of all parts of the connections, from being endangered by a variable impedance in the grid circuit (namely by the coupling with the aerial).

Two stages of low-frequency amplication with a double pentode (ELL 1) follow the detector. One half of the pentode acts as output amplifier and can deliver two watts to a loud speaker connected to it. A loud speaker is used only for calling the car (see below). Two watts are more than enough to make the call tone clearly audible, even when the car is riding through a noisy street. The conversation itself is carried out with an ordinary telemicrophone, the taking up of which automatically switches off the loud speaker [4]).

[3]) See Philips techn. Rev. 3, 248, 1938.

[4]) Since only one definite tone frequency is necessary for calling, a filter may if desired be connected in front of the loud speaker which only passes a narrow frequency range around the call tone frequency. In this way the noise heard in the absence of a signal to be received is very much reduced. Other possibilities of avoiding the noise are for example the use of a call lamp instead of a loud speaker or the short circuiting of the loud speaker by the voltages in the frequency region above 3 500 c/s (wich occur only in the noise and are eliminated during the reception of a signal).

The supply

The cathodes of the valves, which consume a total of about 20 watts, are fed directly from the 6 volt accumulator battery of the car. The anode voltages are obtained from it by means of a vibrator-converter [5]). This contains a vibrating tongue with two pairs of contacts, see *fig. 6*. By means of one

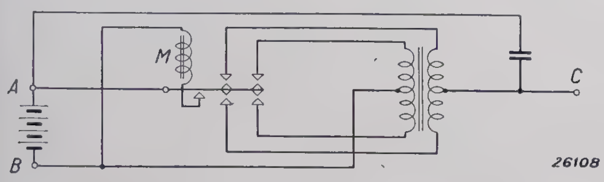


Fig. 6. Diagram of the vibrator. The electromagnet M keeps a vibrating tongue in motion, upon which there are two sets of contacts which simultaneously reverse the current direction in the primary and in the secondary winding of the transformer. After smoothing a D.C. voltage of 250 volts is obtained in this way from the 6-volt accumulator battery AB. This can be taken off between A and C.

set of contacts one of the poles of the accumulator is alternately connected to the ends of a transformer winding, whereupon an A.C. voltage is produced therein. This voltage is transformed up to the desired value and then with the help of the second set of contacts, converted into a (pulsating) D.C. voltage which is afterwards smoothed. Since the power to be dealt with (60 W) is too high for one such vibrator, two vibrators are connected in parallel in the converter.

Since it was desired to house the converter in the same cabinet as the receiver, the avoidance of interferences required special attention. Sparks from the vibrator contacts are eliminated as well as possible by shunting with suitable condensers and resistances; this is also necessary in connection with the wear on the contacts. Furthermore, the whole converter is shielded, and filters are introduced in the connection to the receiver, by means of which the interferences are reduced to a harmless level.

The current in the primary circuit of the converter amounts to 10 A, the connections to the accumulator must therefore be short and thick, and transition resistances in the contacts must be kept small. A resistance of 0.1 ohm, for instance, in this circuit would already cause a fall in the anode voltage of 15 per cent.

At the central station of the service in question the same transmitter and receiver is used as in the cars. The supply, however, can here be taken from the A.C. mains, and the converter replaced by a normal plate voltage apparatus.

[5]) See: J. W. Alexander, Car Radio, Philips techn. Rev. **3**, 112, 1938.

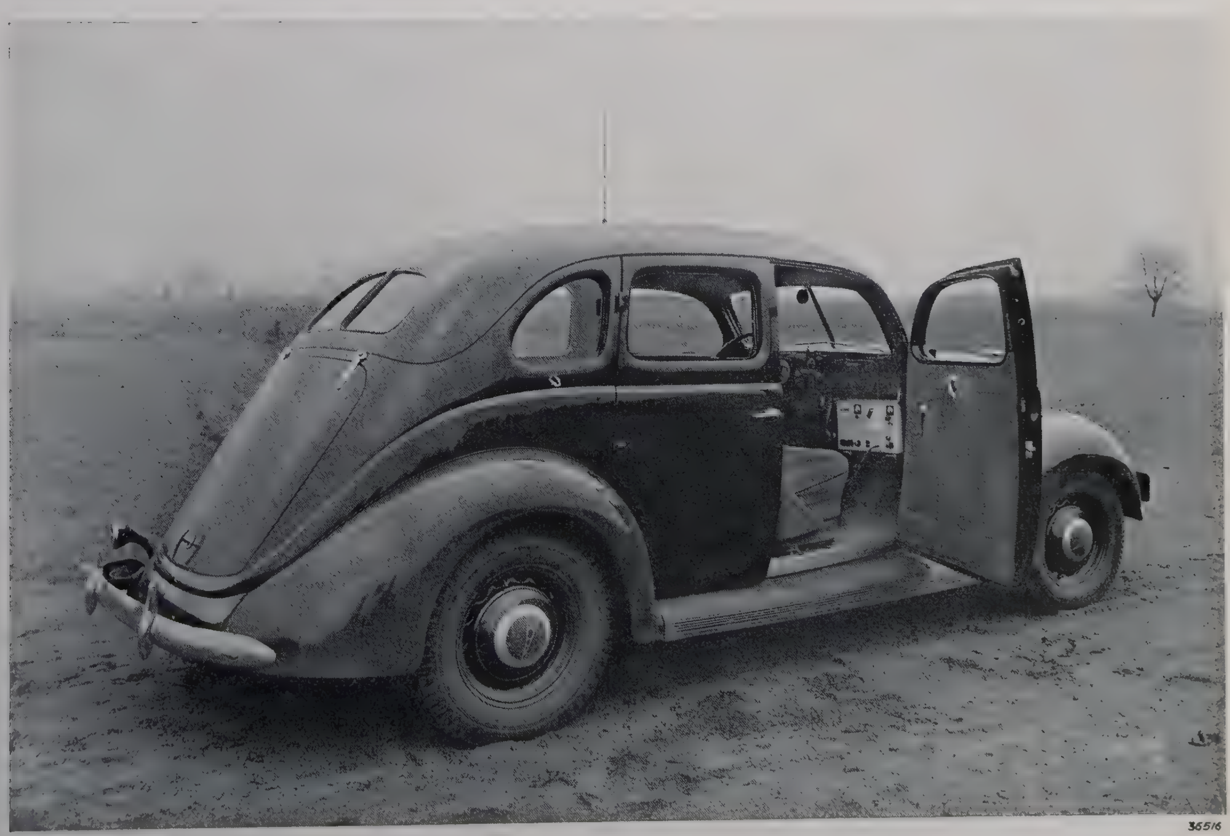


Fig. 7. The aerial rod is screwed into the middle of the roof of the car which acts as counter capacity. Under the dashboard of the car the transmitter receiver DR 38 may be seen.

The aerial

The aerial of the mobile station is a quarter wave length aerial, *i.e.* a vertical rod of about 1 m in length, with the steel top of the car for counter capacity. *Fig. 7* shows how the aerial is installed. In order to prevent damage to the fairly thin top of the car the aerial must be as thin and light as possible, so that upon braking and accelerating and jolting the mass forces occurring are not too great. The rod is best made slightly conical. In order to enter garages and drive under all kinds of obstacles such as low hanging branches, a small spring construction is built into the aerial at the height where it is fastened to the roof of the car.

The aerial is used not only for transmitting but also for receiving. Switching the aerial over to the transmitter or receiver is accomplished by means of the switch altready menioned (see *fig. 8*), which at the same time switches the anode voltage from the transmitter to the receiver, and interrupts the supply of the carbon microphone when receiving. In the third position of the switch the previously mentioned aerial current meter is switched in. In ordinary operation this is short circuited in order to prevent any interruption during transmission due to a possible defect of the thermo element.

In the case of the aerial of the central station the same limitations are not experienced as in the case of the aerials of the cars; an aerial of a half wave length, or, better, several such aerials one above the other, can be used. By using several aerial rods one above the other a concentration in the horizontal plane of the energy radiated is obtained. With three aerial rods, for instance, about the same result is obtained as with a transmitter with three times the power.

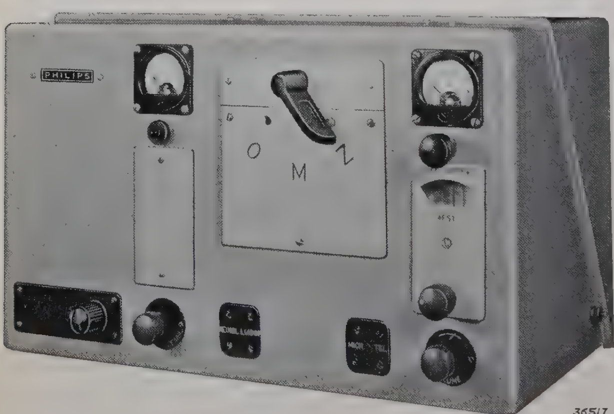


Fig. 8. The transmitter-receiver DR 38. In the middle, the switch with three positions for reception, measuring and transmitting. Below, the connections for the tele-microphone and the call loud speaker.

Use of the installation and results

When the car is moving the switch of the apparatus normally stands at reception, and the tele-microphone lies on its hook, so that the call loud speaker is in connection. If one station, either the central or one of the cars, wishes to make a communication, this station switches over to "transmission" and then transmits the call tone by means of the above-described arrangement in the modulation system. Hereby all the stations of the service are called at the same time and may therefore all receive the communication. In many cases there will be no objection to this, it may even be desirable in order that the different cars may know each others's instructions and whereabouts. If, however, it is desired to be able to call a single car individually, this can be provided for by means of a number choosing arrangement as in the case of the telephone.

Experiments have been carried out with the installation described, with the aerial of the fixed station at a height of 45 m above the ground. Good communication was possible with the mobile station in the moving car at distances of from 10 to 20 km. The greatest distance over which a communication could just be made was 45 km. At this and greater distances, where the field strength becomes very small. stationary waves, which occur upon reflection at all kinds of obstacles in the landscape, begin to play an appreciable part. It is often found in such a case that a displacement of the apparatus by several metres influences the communication considerably. In large cities, as already mentioned in the introduction, the level of interference is of special importance. This is found, however, not to be too great an objection at the frequencies here chosen. Passing cars, however, sometimes give an unpleasant interference which is caused by the sparking plugs. The car in which the apparatus in installed must of course be rendered absolutely free of interference from this source. This is done not only by the usual devices for a car radio receiver, but also by introducing resistances of about 10 000 ohms in series with the sparking plugs.

OPTICAL EXPERIMENTS WITH MODELS ON THE DIRECTIONAL DISTRIBUTION OF SOUND IN HALLS

by R. VERMEULEN.

534.846.6 ; 534.846.3

The sound distribution in a hall can be imitated in a simple way by the light distribution in a small model of the hall. With the help of a small camera obscura the directional distribution of the light can be determined at definite spots in the model, and in this way some information is obtained about the directional distribution of sound in the hall. Serveral records are shown which were made in a model of a broadcasting studio.

In order to study the distribution of the radiation of a source of sound in a hall, use may well be made of small models in which the source of sound is replaced by a light source, and the walls are made of a material which as far as possible reflects the light in the same way as the sound is reflected by the walls in the actual hall.

In a previous article in this periodical it was explained how the acoustics of theatre auditoria can be studied with the help of such models [1]). The principle of the models used for this purpose is given in *fig. 1.* The part of the auditorium in which the audience is seated is left open. This may be done since the sound which is incident on the audience is practically completely absorbed. The light source occupies the position of the stage or of the orchestra.

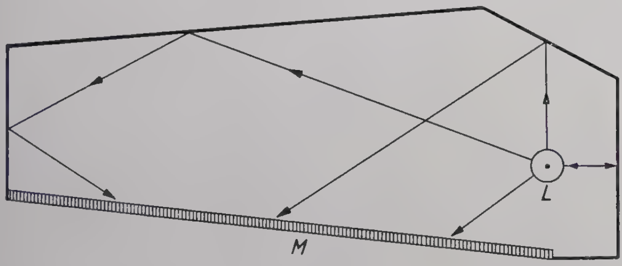


Fig. 1. Optical model for the investigation of the sound distribution in a hall. The source of sound is represented by a lamp *L*. The walls of the model are made of polished aluminium. At the spot where the audience is seated a frosted glass plate *M* is introduced. The light distribution over this plate gives a picture of the sound distribution over the audience.

If a photographic plate or a frosted glass plate is placed over the opening it will receive the direct light from the lamp and the light reflected by the ceiling and walls. The distribution of the intensity of illumination over the floor space will correspond to the distribution of the intensity of sound over the audience. This holds at least as far as the propagation of sound takes place according to the rules of geometric optics. It is true for high tones whose wave length is small compared with the dimensions of the surfaces at which the sound is reflected: for low tones, however, it does not hold. A further restriction of the fidelity of the model lies in the fact that it is almost impossible to make the optical reflection coefficients as large as the acoustic reflection coefficients of many ordinary wall coverings [2]. For this reason there is always a considerable amount of sound radiation in a hall which is due to repeated reflections which are not reproduced by the optical model. This sound radiation has usually covered a fairly long distance and has therefore experienced such a retardation that it no longer contributes to the intelligibility of speech, but is observed as reverberation or echo.

Phenomena of reverberation or echo will therefore often be incapable of being investigated by means of optical models. If we are primarily concerned with intelligibility, however, optical models can be successfully employed. The components with high frequency in the sound, whose propagation obeys the geometric optical laws very well, are of primary importance for the intelligibility [3]). Moreover, only those rays contribute to the intelligibility which are not longer than 20 m from the source to the hearer. These are rays which, in a hall of ordinary dimensions, will usually only have undergone one or two reflections [3]), and in the case of which the too great absorption in the

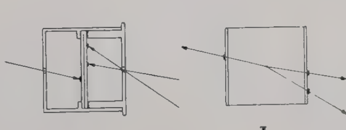


Fig. 2. *a*) Camera obscura (actual size) for the determination of the directional distribution of light at a given spot in a room.
b) The photographs obtained, pasted to a cube of the same size as the camera, give a clear picture of the directional distribution.

[1]) Philips techn. Rev. **1**, 46, 1936.

[2]) The models were made of aluminium which when entirely clean and polished has an optical absorption coefficient of about 15 per cent. This is about equal to the acoustic absorption coefficient of a carpet, while a hard wall has an absorption of only about 3 per cent.

[3]) See on this subject the article: Auditorium acoustics and intelligibility, Philips techn. Rev. **3**, 139, 1938.

optical model does not yet have a disturbing effect. For this so-called effective sound it may therefore be stated, that not only the intensity but also the directional distribution of the radiation corresponds at every point in the hall with the intensity and directional distribution of the light at the corresponding point in the model.

The previously described experiments only provide information about the total sound intensity at a given spot, but not about the direction from which the sound is incident. In the investigation of theatre auditoria this latter information is not usually required; it is, however, required in the investigation of broadcasting studios, where it is desired to know not only the intensity but also the directional distribution at the point occupied by the microphone.

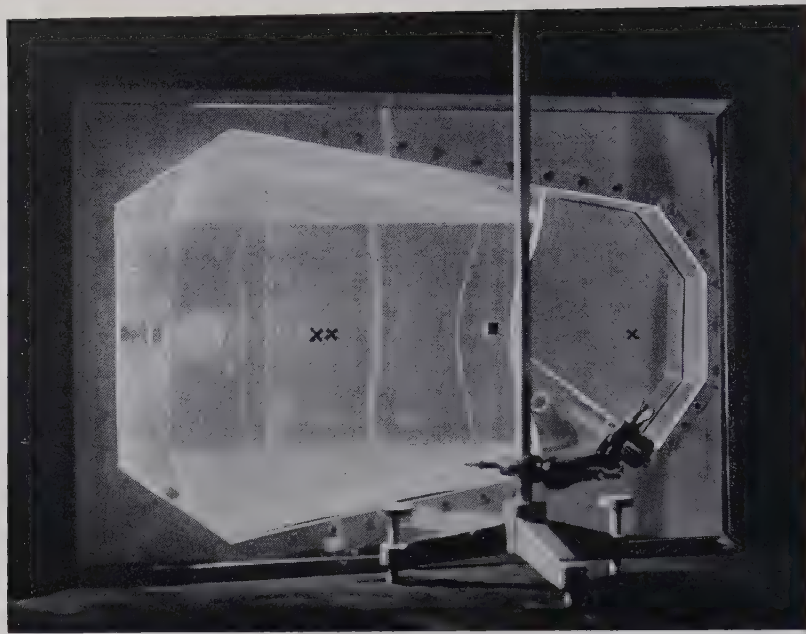


Fig. 3. Measurement with the camera obscura at a given spot in the model of a hall. Three other measuring spots are indicated by crosses.

In order to be able to determine the directional distribution of the radiation which is incident upon a given point, a "camera obscura" was constructed (see *fig. 2*). This has the form of a cube with an edge of 1 cm. At the centres of two opposite sides small holes are bored, while halfway between these holes a piece of photographic paper is inserted which is light-sensitive on both sides. When this camera is placed at a spot in the model at which one desires to investigate the directional distribution of the sound (see *fig. 3*), an image will be formed on both sides of the photographic paper from which it can be judged from which main direction the light was incident upon the front and rear of the camera obscura.

The acceptance of the camera obscura to the front and rear side amounts to only about 90°. By making three exposures with the holes of the camera first front and back, then right and left and finally top and bottom, a complete picture can be obtained of the directional distribution of the sound at the spot investigated.

This picture can be made very clear by pasting the photographs obtained upon a cube so that the line joining the centre of the cube and a blackened point of a photograph pasted on one side corresponds to the direction of the ray which caused the blackening of that point (fig. 2*b*).

Experiments were done with a model of a broadcasting studio which is shown in *fig. 4*. The arrangement shown in fig. 3 also refers to this studio. The directional distribution of the sound (the light) was determined at four points in the room which may be considered as positions for the microphone.

In the cross section sketch the path of several rays is given between the orchestra (the lamp) and the microphone (the camera obscura). It may be seen how the different facets of the walls and the ceiling above the orchestra contribute to the concentration of the sound rays and serve to send the sound toward the back of the hall. At the same time from this drawing it may be seen from which direction the sound rays reflected by the different walls are incident upon the microphone. These directions are also indicated for the three other measuring points, but without the reproduction of the entire path of the rays.

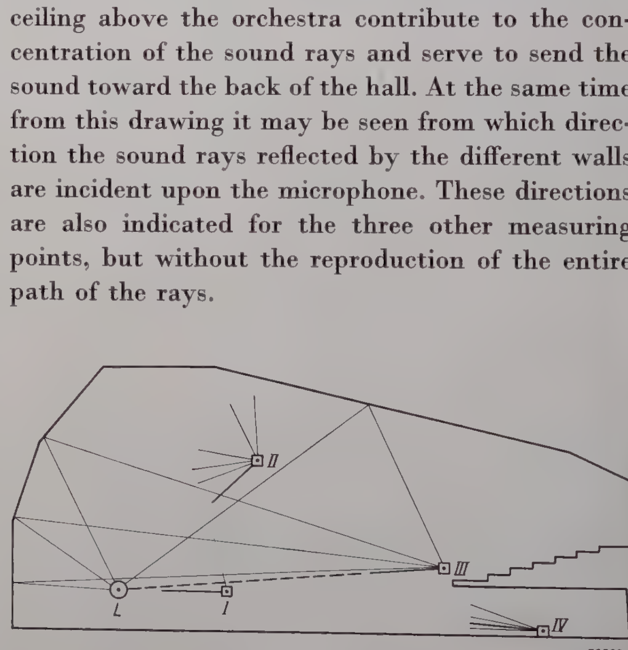
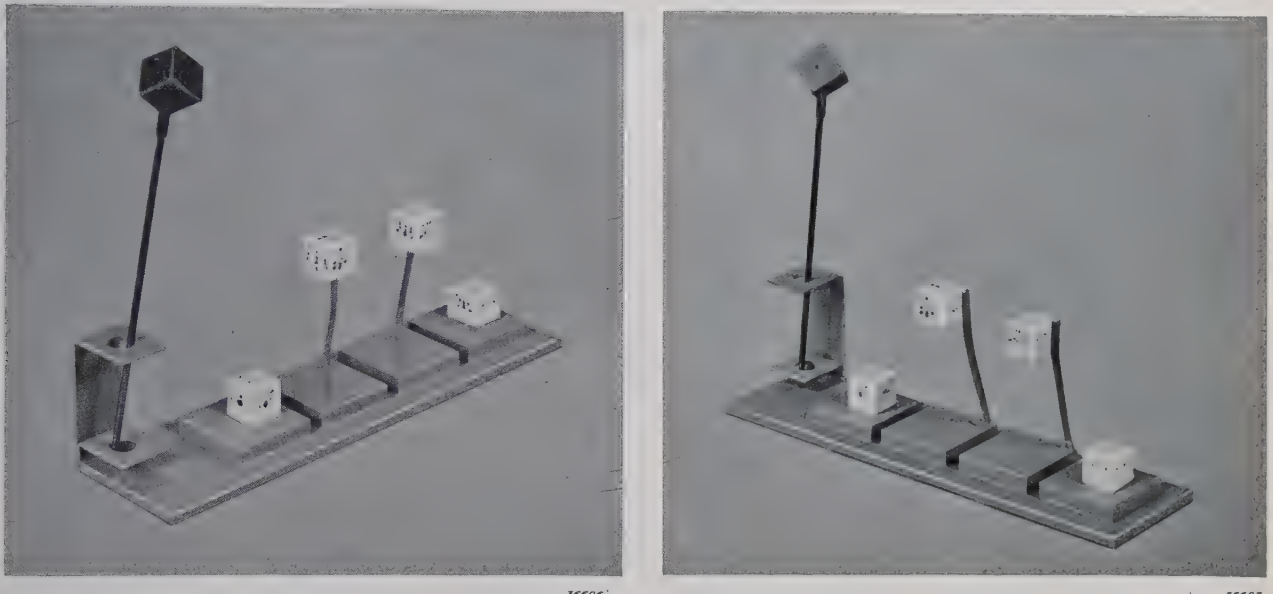


Fig. 4. Model of a broadcasting studio in Hilversum (Netherlands). *L* lamp; *I*, *II*, *III*, *IV* positions where the directional distribution of the sound was investigated with the help of a camera obscura. For position *III* the possible sound rays are drawn from the lamp to the measuring point. For the other positions these rays are indicated by short lines.

The photographs obtained with the camera, pasted to a cube in the manner described, are shown in *fig. 5a* and *b*. Most of the ray directions indicated in fig. 4 can easily be found in fig. 5. In addition to points which correspond to these ray directions, however, numerous other spots may be seen, which must be ascribed to reflections by the oblique surfaces which are situated between ceiling and walls (see fig. 3). The high intensity of these spots indicates that by a suitable construction of these oblique surfaces a considerable amplification of the sound can be obtained at the back of the hall.

In fig. 5*b* where the cubes are seen from the rear, it is striking that it is the cubes placed toward the front of the model which exhibit an appreciable blackening on their rear sides. Since the sound contribution corresponding to this covers a considerably longer path than the direct sound in the front of the hall, it might lead to disturbing echo phenomena. For this reason the rear wall of the hall was covered with a material which damps the sound very much.



a *b*

Fig. 5. From left to right: records of the directional distribution of the sound at the positions *I-IV* of fig. 4 pasted to cubes according to the method given in fig. 2. *a*) gives the front and *b*) the rear side. From the photograph of the rear side it may be seen that a considerable amount of sound energy is reflected by the rear wall to the measuring points *I* and *II*, which in the actual hall would lead to echoes. In the stand may be seen the camera fastened to an axis in the direction of a cube diagonal. Simply by rotating this axis the camera can be brought successively into the three desired positions for the three exposures.

GEOMETRICAL CONFIGURATIONS AND DUALITY OF ELECTRICAL NETWORKS

by B. D. H. TELLEGEN.

513.84 : 621.392

Two networks may have such a relation to each other that one behaves with respect to the current flowing in it in a way entirely analogous to the way in which the other behaves with respect to the voltages prevailing in it. Such a "duality" between two networks can, however, only exists when the geometrical configuration according to which the networks are built up satisfies the condition that it can be drawn in one plane without two branches crossing each other. After a consideration of the possible geometrical configurations, this condition is illustrated by means of examples.

In calculations of electrical networks analogous formulae are often obtained when in one network the currents are calculated and in another the voltages. A simple example of two such networks is shown in *figs. 1a* and *b*. In order to present the analogy mentioned as clearly as possible we shall introduce the following concepts: an arbitrary electrical element (resistance, coil, condenser, source of voltage, source of current, etc.) we shall call a branch of the network; the end of a branch, which at the same time forms the connecting point of two or more branches, we shall call a junction; a closed circuit, which is formed by any arbitrarily chosen branches of the network, will be called a mesh.

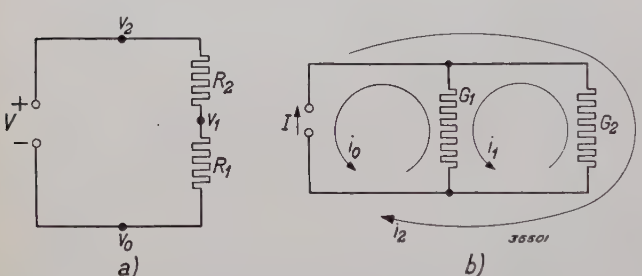


Fig. 1. Two simple networks which are dual with respect to each other.

The following statements may then be made about the network of fig. 1*a*.

A source of voltage V is applied to two resistances R_1 and R_2 in series. The current through the resistances is:

$$I = \frac{V}{R_1 + R_2} \cdot$$

The voltages V_1 and V_2 on R_1 and R_2 are:

$$V_1 = \frac{R_1}{R_1 + R_2} V, \qquad V_2 = \frac{R_2}{R_1 + R_2} V.$$

The system contains three branches and three junctions. The voltages on the three branches are in each case equal to the differences between the potentials of the junctions, v_0, v_1 and v_2:

$$v_1 - v_0 = V_1, \quad v_2 - v_1 = V_2, \quad v_2 - v_0 = V.$$

Since the voltages on the branches do not change when the potentials of all the junctions are increased by the same amount, the latter are determined except for an additive constant.

Correspondingly the following may be stated about the network of fig. 1*b*.

A source of current I is applied to two conductances G_1 and G_2 in parallel. The voltage on the conductances is:

$$V = \frac{I}{G_1 + G_2} \cdot$$

The currents I_1 and I_2 through G_1 and G_2 are:

$$I_1 = \frac{G_1}{G_1 + G_2} I, \qquad I_2 = \frac{G_2}{G_1 + G_2} I.$$

The system contains three branches and three meshes. The currents through the three branches are in each case equal to the differences between two of the mesh currents i_0, i_1 and i_2:

$$i_1 - i_0 = I_1, \quad i_2 - i_1 = I_2, \quad i_2 - i_0 = I.$$

Since the currents through the branches do not change when all the mesh currents are increased by the same amount, the latter are determined except for an additive constant [1]).

If we compare the statements about the two systems we see that they become identical when we interchange the words voltage and current, resistance and conductance, connection in series and connection in parallel, junction and mesh. Networks which also contain capacities and self-inductions can in the same way be compared with each other by interchanging the words capacity and self-induction. Networks which can be compared in this way are said to be dual with respect to each other [2]). If one has studied the properties of one network, the properties of the dual network are thereby also known.

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[1]) In the customary method of calculating with mesh currents, one of these is usually set equal to zero. For the purpose in view, however, this cannot be done here.

[2]) Instead of dual the words reciprocal or inverse are also used.

The purpose of this article is to show how we can generalize the duality found in the example of fig. 1 in order to be able to indicate a dual system in the case of networks of more complex composition also.

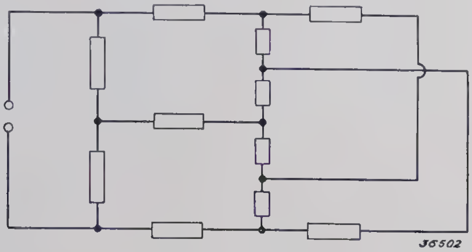

Fig. 2. Example of a network for which no dual system can be found.

We shall not consider mutual inductions in doing this, although it is possible to include them in the discussion. It will be found that this generalization is not always possible, in other words, that networks exist which have no dual system. An example of such a network is drawn in *fig. 2*. The conditions which a network must satisfy in order to have a dual system can be found by the help of topological considerations of the network. One is thereby not concerned with the electrical significance of the branches occurring in the network, but with the geometrical configuration of the network, in which it is only the way in which the junctions are connected to each other, and not the position of the junctions and the length and form of the branches which is significant. The branch of geometry which considers figures in this way is analysis situs or topology, and the configurations in question are known as graphs.

Geometrical configurations of the networks

We shall begin with a general investigation of the geometrical configurations of the networks [3]). In the case of networks which have pairs of terminals, we consider each pair of terminals as a branch of the configuration. If we are, for example, concerned with a quadripole, such as is represented in *fig. 3a*, consisting of two pairs of terminals connected by three resistances in star connection, the system contains four junctions connected by five branches, and we may draw the configuration of this quadripole as indicated in fig. 3*b*. Due to the fact that we consider the pairs of terminals as branches, the currents flow entirely within the configuration. In this way we can leave out of consideration configurations which consist of two parts which are either unconnected or connected at only one point, since in such configurations no current can flow from the one part of the configuration to the other, and the two parts are therefore entirely independent of each other electrically.

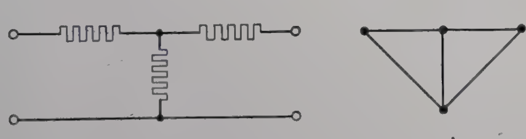

Fig. 3. A network with two pairs of terminals (*a*) and its configuration (*b*).

[3]) R. M. Foster, Geometrical circuits of electrical networks, Trans. A.I.E.E. **51**, 309. 1932.

We now draw successively all the configurations which are possible with a given number of branches. This is done in *fig. 4* for up to and including seven branches. With two branches, only one configuration is possible. With three branches two configurations are obtained, with four branches five, with five there are six, with six branches 13, with seven branches 28, etc. The two configurations with three branches are formed from the configuration with two branches, by replacing one branch in the latter by two branches in parallel or two branches in series. In the same way all the configurations with four branches can be derived from the configurations with three branches, and similarly all the configurations with five branches, from those with four branches. This can be easily verified by means of fig. 4. In the case of the configurations with six branches, however, this derivation is no longer entirely valid. Among these configurations there is one (indicated in fig. 4 with heavy lines) in which no single pair of branches is in series or in parallel, so that this configuration cannot therefore be derived in the manner described from a configuration with one branch less. The configurations with seven branches can again all be derived from those with six branches, whereby two configurations with seven branches are formed from the configuration with six branches printed with heavy lines.

Since the number of configurations with more than seven branches becomes very large, in drawing them we shall confine ourselves to those configurations which contain no branches in series or in parallel, since the others can always be derived in the manner described from the configurations with one branch less. Configurations without connections in series or in parallel are only possible with definite numbers t and k of branches and junctions, as will appear from the following.

If a configuration contains no branches in series, then at least 3 branches come together at every junction. If we count the number of branches coming together at every junction and multiply by the

number of junctions, we arrive at the total of at least $3k$ branches. Every branch is hereby counted twice, so that

$$t \geqq 3k/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If a configuration contains no branches in parallel, it cannot contain more branches than when each junction is connected to every other junction by one branch. At every junction, therefore, not more than k—1 branches come together. If for all the junctions we count the number of branches coming together at that point, we therefore arrive at the total of not more than $k(k-1)$ branches, whereby every branch is again counted twice, so that

$$t \leqq k(k-1)/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2) it follows that $k\ (k-1) \geqq 3k$, and therefore $k \geqq 4$, from which with (1) it follows that $t \geqq 6$. Actually, among the configurations reproduced in fig. 4, it was with six branches that we first found a configuration which contains no connections in series or in parallel. The values of k which are possible at certain values of t on the basis of (1) and (2) are given by the following table:

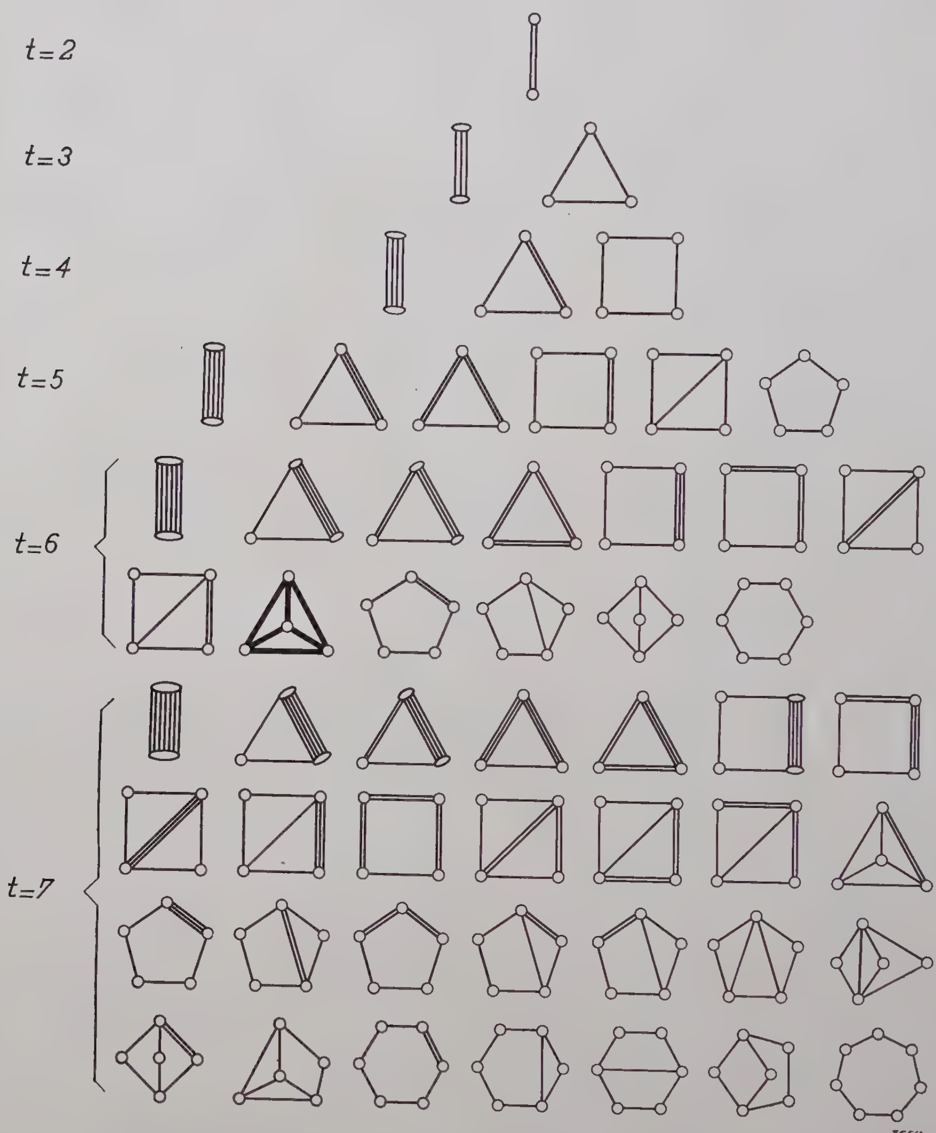

Fig. 4. Possible configurations with t branches, for t up to and including 7. The heavily printed configuration in the series with $t = 6$ differs from the others by the fact that it contains no connections in parallel or in series.

t =	6	7	8	9	10	11	12
k =	4	—	5	5 6	5 6	6 7	6 7 8

In *fig. 5* these configurations are drawn for up to and including 11 branches.

The configuration with six branches which already occurs in fig. 4 is that of **Wheatstone's** bridge. All the branches and junctions in it are equivalent, which can best be seen when the figure is imagined as a tetrahedron (*fig. 6a*). In a corresponding way the configuration with $t = 8$ can be imagined as a square pyramid (fig. 6*b*), that with $t = 9, k = 5$ as two tetrahedrons base to base (fig. 6*c*).

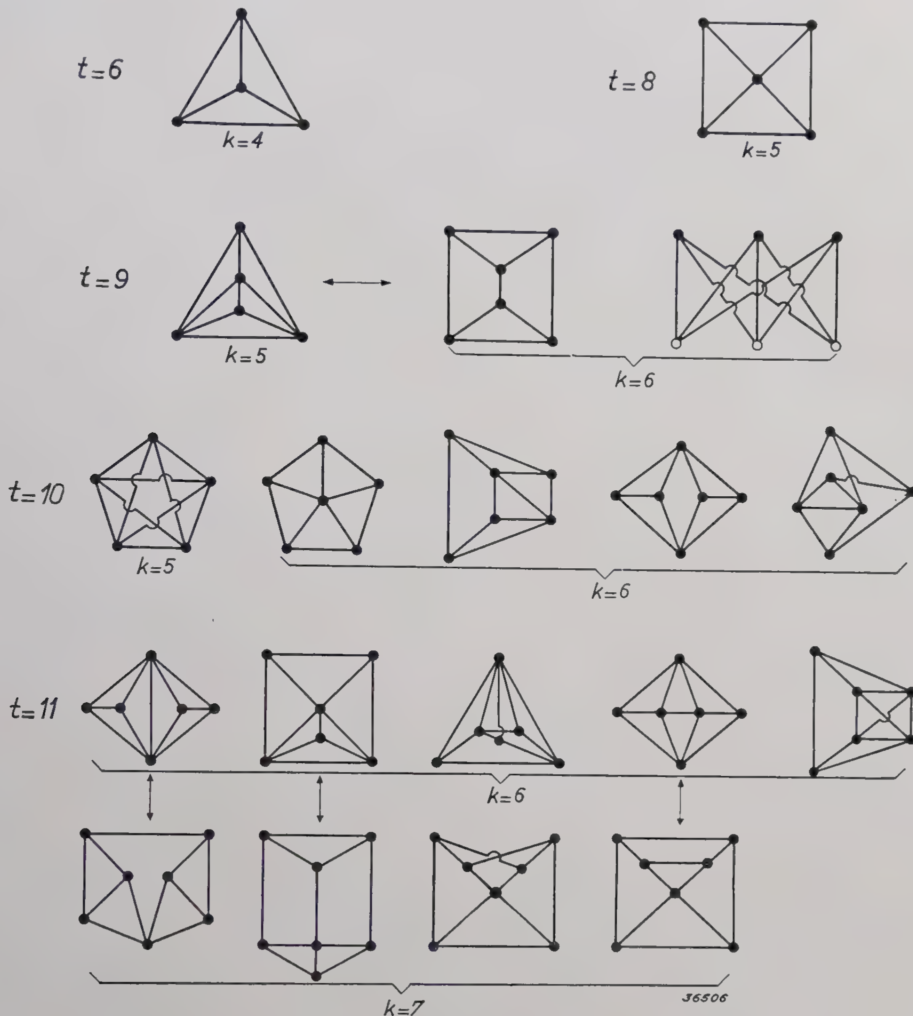

Fig. 5. Configurations for t up to and including 11, in which no connections of branches in parallel or in series occur. All planar configurations (see below) are drawn without crossings. The configurations connected by arrows are dual with respect to each other, the other planar configurations are dual in themselves.

With $t = 9$, $k = 6$ two configurations are possible, the first of which can be conceived as a triangular prism (fig. 6*d*). In the case of the second, which consists of two sets of three junctions, with each junction of one set connected to each junction of the other set, such an interpretation as a simple solid (a polyhedron) is, however, no longer possible, and the same is true of the configuration with $t = 10$, $k = 5$, in which each junction is connected to every other junction (the complete pentagon). Since these facts are of decisive importance for the duality, we shall subject them to a closer examination.

The two last-mentioned configurations, which we shall indicate by A and B in the following, and which are again shown separately in *fig. 7*, cannot be drawn in a plane without at least two branches crossing each other. This can be easily verified by attempting to do so, and it can also be proved directly (see below). Configurations which can be drawn in a plane without crossed branches we shall call **planar**; A and B are therefore not planar. It can be shown that all non-planar configurations can be reduced to the opening or short circuiting of branches of A and B, so that A and B are the fundamental types of non-planar configurations [4]).

[4]) C. **Kuratowski**, Sur le problème des courbes gauches en topologie, Fundamenta Math. **15**, 271, 1930. H. **Whitney**, Planar graphs, Fundamenta Math. **21**, 73, 1933.

The fact that A and B are not planar is related to the above-mentioned fact that it is impossible to conceive the branches of these configurations as the edges of a polyhedron. This may be understood

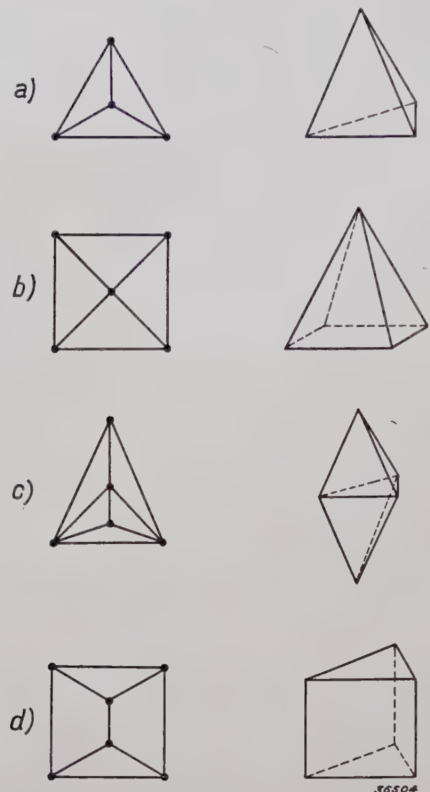

Fig. 6. Certain configurations can be imagined as polyhedrons: *a*) tetrahedron, $t = 6$, $k = 4$; *b*) square pyramid, $t = 8$, $k = 5$; double tetrahedron, $t = 9$, $k = 5$, *d*) triangular prism, $t = 9$, $k = 6$.

as follows. A planar configuration can also be drawn on a sphere without two branches crossing each other. The surface of the sphere is then divided by the configuration into a number of side surfaces, so that the part of the plane originally surrounding the configuration also becomes a side surface of the sphere. Since a sphere and a polyhedron are topologically equivalent, we may also consider the configuration drawn on the sphere as a polyhedron, and we may imagine the polyhedrons given in fig. 6 as being formed in this way from the corresponding configurations. Conversely every configuration drawn on a sphere without crossing can also be drawn without crossing in a plane [5]).

The boundary of every side surface in which a sphere is divided by a configuration drawn upon it, forms a mesh in the sense defined at the beginning. Between the number m of these side surfaces or meshes, the number t of the branches and the number k of the junctions there is a relation, called the *Euler polyhedron formula*, of the following form:

$$k - t + m = 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

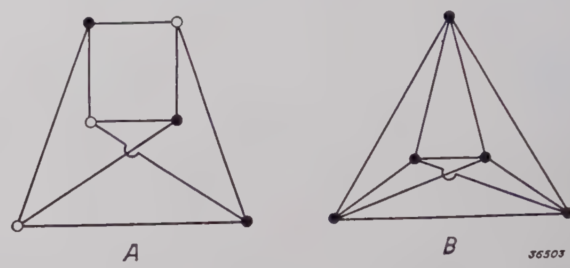

Fig. 7. The two fundamental types A and B of all non-planar configurations. The configuration A, with $t = 9$, $k = 6$ is built up of three sets of three junctions, each junction of one set (circles) is connected to each junction (points) of the other set. The configuration B with $t = 10$, $k = 5$, consists of five junctions, each of which is connected to every other. Both configurations. which occurred also in fig. 5, are here so drawn that there is only one point where two branches cross.

With the help of this formula we can demonstrate the lack of planarity of the configurations A and B. Assume that A (with $t = 9$, $k = 6$) is planar. Drawn on the sphere, it must be true that $m = t - k + 2 = 5$. Since every branch must belong to two of these meshes, $2t = 18$ branches are available for these meshes. However, A contains no triangles, so that for the meshes at least $5 \times 4 = 20$ branches are necessary, thus more than those available. From this it follows that A cannot be planar. Similar reasoning can be applied in the case of B [6]).

A and B can be drawn on other surfaces [7]) without crossing, since for such surfaces $k - t + m < 2$ is always true. For a torus for example $k - t + m = 0$. A may be drawn on a torus as is done in *fig. 8*; the torus is thereby divided into $m = 9 - 6 = 3$ meshes. B can also be drawn on a torus, in two ways, as indicated in *fig. 8*. In both cases the torus is divided into 5 meshes. Another type of surface is Möbius' strip which is obtained by giving a half turn to an open strip and then joining the ends. On this surface, starting from any point and passing around the strip, we can reach the corresponding point on the other side of the strip. Such surfaces are called unilateral surfaces. For Möbius' strip $k - t + m = 1$. The configurations A and B can also be drawn upon it, as indicated in *fig. 9* and B can again be drawn in two ways.

Conditions for duality and construction of the dual network

When generalizing the duality found in the case

[5]) Each side surface of the sphere may be transformed into the part of the plane surrounding the configuration. Drawn on the sphere, the configuration with eight branches, for example, from fig. 5, contains four triangles and one quadrangle. In fig. 5 it is so drawn in a plane that the quadrangle forms the periphery. It may, however, also be drawn so that one of the trangles forms the periphery.

[6]) F. Levi, Streckenkomplexe auf Flächen, Math. Z. **16**, 148, 1923.

[7]) I. N. Kagno, The mapping of graphs on surfaces. J. Math. and Phys. **16**, 46, 1937.

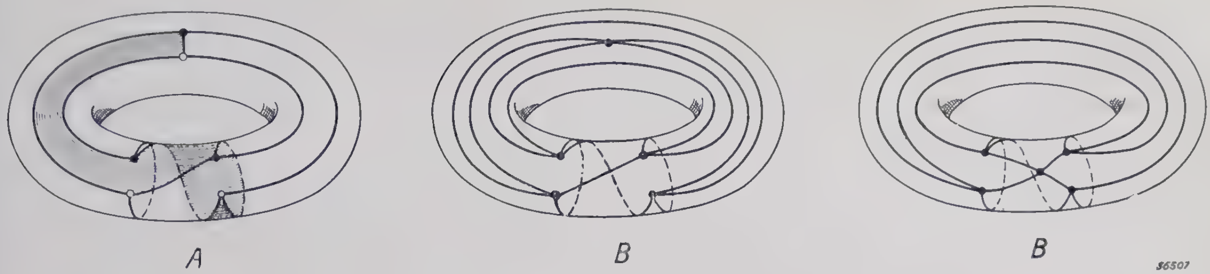

Fig. 8. The non-planar configurations A and B drawn on a torus. Configuration A divides the torus into 3 meshes, each of 6 branches. One of the side surfaces which are bounded by these meshes is here indicated by shading. Configuration B divides the torus either into 2 meshes of 5, 1 mesh of 4 and 2 meshes of 3 branches, or into 5 meshes of 4 branches.

of the example in fig. 1 we begin with the requirement that the junctions of the dual system shall correspond to certain meshes of the original system. This leads to the following result [8]):

A network has a dual system when and only when it is planar.

The network of fig. 2 is not planar, since it can be derived from the configuration A, as is shown in *fig. 10*, when the branch indicated is opened and one each of the resulting pairs of branches in series is short circuited. The network therefore has no dual system.

If a network is planar, the dual system can be found in the following way. If we draw the configuration of the network without crossings on a sphere (or in a plane) certain meshes are emphasized, namely the meshes which bound the side surfaces of the sphere. These meshes must now correspond to the junctions of the dual system. Within each of these meshes (in the plane, outside the outermost mesh also) we therefore assume a new junction and connect every pair of these new junctions which lie in adjacent meshes by a new branch, which crosses the common branch of these meshes (see *fig. 11*). If the meshes have more than one branch in common, the corresponding new junctions are also joined by more than one branch. We arrive in this way at a new planar configuration which is the dual of the original, and in which each of the original junctions lies in a mesh of the new system. *The duality is thus reciprocal.* In fig. 5 arrows indicate which configurations are dual with respect to each other. The other planar configurations in this figure are dual in themselves.

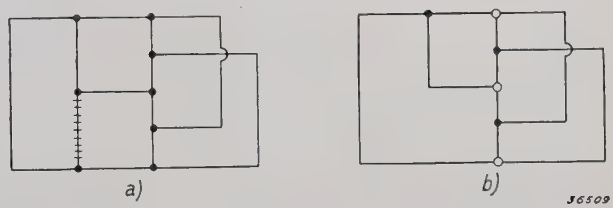

Fig. 10. The network reproduced in fig. 2 can be converted into configuration A by opening one branch and short circuiting two others.

In order to provide that to every equation which can be written for the original system there is a corresponding, entirely analogous equation for the dual system, we must, in the manner indicated at the beginning, change the electrical quantities in the original system into corresponding quantities of the dual system. A current I, for instance, becomes a voltage V. Since these quantities have

[8]) H. Whitney, Non-separable and planar graphs, Trans. Amer. Math. Soc. **34**, 339, 1932. Since the derivation of the theorem is rather elaborate and involves abstract considerations it is not given here; it will be published in T. Ned. Rad.-Genoot.

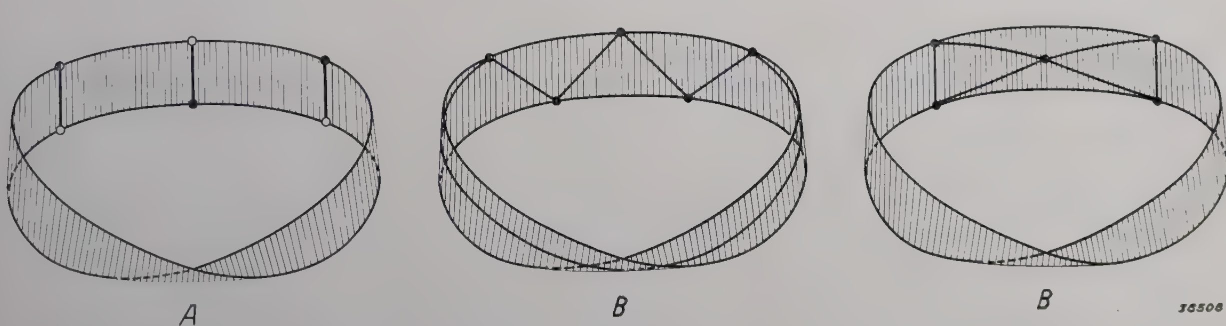

Fig. 9. The non-planar configurations A and B drawn on Möbius' strip. On this surface A forms 3 meshes of 4 and one of 6 branches, B forms either 5 meshes of 3 and 1 of 5 branches, or 4 meshes of 3 and 2 of 4 branches.

different dimensions we may not immediately set them equal to each other, but we must introduce a dimension factor K which has the dimension of a resistance, and whose magnitude we may choose arbitrarily. When we have made $V = KI$ in this way, then the product of every pair of impedances of the two dual networks compared with each other is equal to K^2.

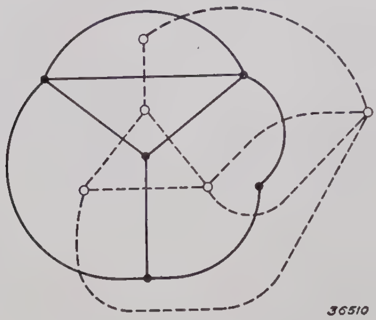

Fig. 11. Construction of the dual configuration (broken lines) for a given configuration (full lines).

If the system contains connections in series, for instance a self-induction, a resistance, a capacity and a source of voltage in series, these must be considered as four branches which are therefore replaced in the dual system by a capacity, a conductance, a self induction and a source of current connected in parallel.

As an example let us consider the oscillation circuits given in *fig. 12a*, which are coupled over a self induction, and the first of which contains a source of voltage. The dual system is drawn in fig. 12*b*. If we assume in the first system two mesh currents i_1 and i_2, the equations for these meshes become:

$$\left(\frac{1}{j\omega C} + R + j\omega L_1\right) i_1 + j\omega L_2\,(i_1 - i_2) = V,$$

$$\left(\frac{1}{j\omega C} + R + j\omega L_1\right) i_2 + j\omega L_2\,(i_2 - i_1) = 0.$$

If in these we replace the quantities of the first system by the quantities of the dual system in accordance with the above, be replacing C by L/K^2, L_1 by K^2C_1, L_2 by K^2C_2, R by K^2G, V by KI, i_1 by v_1/K, i_2 by v_2/K, where v_1 and v_2 are the voltages on the circuits of the dual system, the above mesh equations pass over into the following equations:

$$\left(\frac{1}{j\omega L} + G + j\omega C_1\right) v_1 + j\omega C_2\,(v_1 - v_2) = I,$$

$$\left(\frac{1}{j\omega L} + G + j\omega C_1\right) v_2 + j\omega C_2\,(v_2 - v_1) = 0.$$

These are actually the junction equations of the dual system.

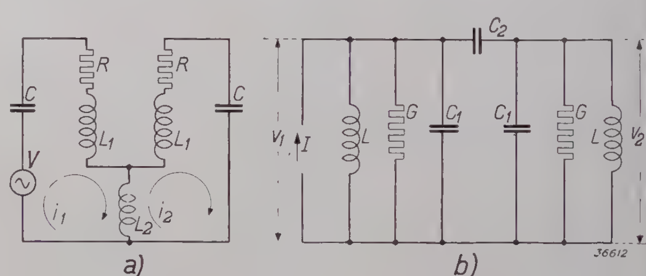

Fig. 12. Example of a network (*a*) for which the dual network (*b*) has been constructed according to the directions given in the text.

AN APPARATUS FOR THE MEASUREMENT OF THE PHOTOGRAPHIC DENSITY OF FILMS

by J. M. LEDEBOER.

771.534.531.5

In measuring photographic density different values can be obtained according to the apparatus used. In the case of the photographic measurement of radiation this is without effect as long as a given arrangement is used. For the recording of density curves of photographic films or plates in order to judge the qality of the picture, however, the arrangement must be adapted to the method of observing (or copying) the film or plate. An apparatus is described which is intended particularly for density measurements of X-ray films. The density can be read off directly on a scale. By the application of an amplifier with automatic control a linear density scale is obtained.

The quality of a photograph, whether a simple amateur photograph or an X-ray photograph for medical or industrial purposes, with given exposure conditions, is mainly determined by the density curve of the photographic film (or plate), *i.e.* by the curve which indicates the relation between the intensity of the radiation incident on the film and the blackening caused by it. In order to judge the value of a film (or of the method of development employed), therefore, this curve must be known. In other cases also, where it is not a question of the quality of the picture, a knowledge of the density curve is required, especially when use is made of the blackening of the film in order to determine quantitatively radiation intensities. Examples of this may be found in spectrography (intensity distribution in spectra), in astronomy (magnitude of stars) and in many other branches of applied physics. Less familiar applications are the determination of the distribution of brightness on road surfaces [1]) and the control of X-ray dosages [2]).

Not only in these measurements themselves, but also in recording the density curve which is required in these cases for calibration, and in general for the estimation of film quality, the problem is encountered of determining the density of a film. We shall here explain briefly how this can be done and then describe a simple, directly indicating instrument for such measurements.

Definition of the density and principle of its measurement

If an amount of light I_0 is allowed to fall upon a blackened film, only a part I of this light will be transmitted. The density of the film is then

$$Z = {}^{10}\log I_0/I.$$

This definition gives some idea of what is meant by "density"; it is, however, inadequate for a quantitative measurement. The value obtained according to the definition by the measurement of I_o and I will still depend very much upon the way in which the light is made to fall upon the film being investigated, and upon the way in which the light is captured which has been transmitted by the film. The loss of light in the blackened film is caused partly by absorption and partly by reflection and lateral scattering [3]). If for example a light ray (I_0) is allowed to fall perpendicularly upon the film and the light emerging at the back in the same direction as the ray is measured (*fig. 1a*), then for that direc-

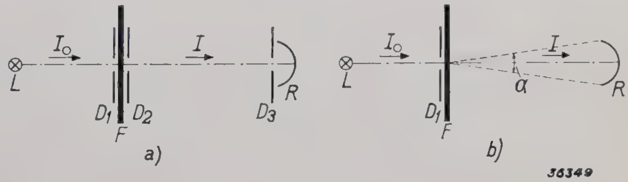

Fig. 1. Arrangements for density measurements. The source of light L radiates on the blackened film F a quantity of light I_0, only part of which I passes through to the receiver R. In *a*) R receives only the direct ray because of the diaphragms D_1-D_3, in *b*); it receives in addition all the rays scattered within the angle α.

tion the absorbed and reflected light as well as the scattered light is lost. If on the other hand the light emerging within a certain angle is measured (fig. 1*b*) — and this is practically always the case, thanks to the finite aperture of the receiver of the light — then part of the scattered light is also measured, and therefore a lower density is found. In *fig. 2a* this effect is given for different densities.

The absorption in the blackened film will be greater for a light ray which passes obliquely through the film than for a perpendicular ray. If the light is incident, not in a parallel beam, but dif-

[1]) See P. J. Bouma. Measurements carried out on road lighting systems already installed, Philips techn. Rev. **4**, 292, 1939, especially p. 294-5.

[2]) See A. Bouwers and J. H. van der Tuuk, Fortschr. Röntgenstr. **41**, 767, 1930.

[3]) The considerations in this section are borrowed from: J. E. de Graaf, Zur Densitometrie von Röntgenfilmen und ihrer Normung. Z. wiss. Photogr. **37**, 147, 1938.

fusely upon the film to be investigated, then, due to this effect, a smaller value of I/I_0, and thus a higher density, may be found with a greater aperture of the receiver, since in this case the rays incident upon the receiver pass more obliquely through the film and are thus more attenuated.

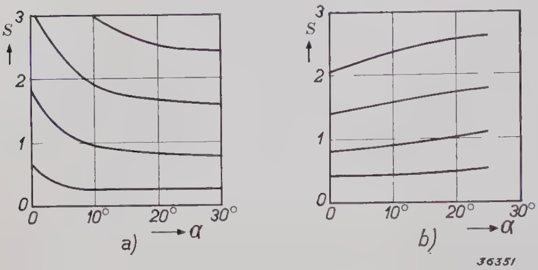

Fig. 2. Density value Z which is found in measuring the same film with receivers of different acceptance angles α (see fig. 1*b*) according to de Graaf [3]). The four curves were obtained with four different heavily blackened films. *a*) A parallel beam was incident on the film. *b*) The film was diffusely illuminated.

This is shown clearly in fig. 2*b*. For a quantitative explanation of the curves there shown the relation between absorption and ray direction as well as the effect of scattering represented in fig. 2*a* must be taken into account.

Finally the colour of the light used for the measurement and the spectral sensitivity of the receiver will affect the value of I_0 found, since the scattering by the grains of the blackened layer depends upon the wave length.

If the purpose of the density measurement is the determination of radiation intensities, it is of little importance what colour of light or direction of beam one uses, if the density measurement is carried out in the same way in calibration and in the measurement itself. The density is in this case only an intermediate quantity. If the density measurement is for the purpose of judging the photographic quality of a film or plate, however, it must be kept in mind that the density differences are the component element of the picture to be obtained, and thus that the manner in which the eye (or the copying material) registers the density also plays a part in the quality which may be assigned to the picture. In this case it is necessary to adapt the density measurement to the manner of observation of the picture (or to the method of copying).

The apparatus which we shall describe in the following was especially developed for the measurement of the density of X-ray films. X-ray films are usually examined by transmitted light with the help of a light box, *i.e.* with diffuse illumination. The eye of the observer is at a distance of about 30 cm from the film. The diameter of the opening of the "receiver", *i.e.* the diamter of the pupil of the eye, is about $^1/_2$ cm. We have therefore chosen exactly the same dimensions for our apparatus (*fig. 3*). As receiver a photocell was used. The spectral sensitivity of this cell should actually be matched to the spectral eye sensitivity; in our case, however, this was unnecessary, since according to a proposal of de Graaf [3]) we used the practically monochromatic light of a sodium lamp for the measurements. For the observation of X-ray films with the light box the use of sodium light is not yet generally customary, it is, however, to be recommended in connection with the relatively slight fatique experienced by the eye upon long continued observation with sodium light.

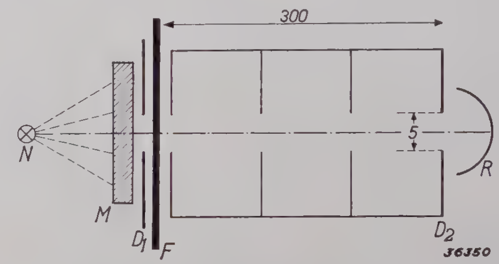

Fig. 3. Arrangement for density measurement of X-ray films, adapted to the method by which the films are examined with the light box. N sodium lamp, M frosted glass, D_1 diaphragm, F film. The aperture of the photocell R is reduced by means of the diaphragm, D_2 to the diameter of the pupil of the eye (5 mm), the distance F-D_2 is equal to the ordinary distance between eye and film (300 mm).

Construction of the apparatus

Arrangement for direct indication

The sodium lamp used in the apparatus burns on alternating current (50 c/s), so that a light flux is obtained which alternates at 100 c/s. The photocell serving as receiver for I therefore gives an alternating current of the same frequency. This is amplified in a two-stage A.C. amplifier, and after rectification the output signal is measured with an ordinary voltmeter (see the diagram of *fig. 4*). The meter can be calibrated directly in density values when care is taken that the extremities of the scale always correspond to the value $Z = 0$ and $Z = \infty$, respectively. For $Z = \infty$, $I = 0$, and this is therefore the pointer indication with no input signal. The pointer can be set accurately on the line for ∞ of the scale by means of a setting screw on the meter. For $Z = 0$, *i.e.* $I = I_0$, the pointer, which here has its greatest deviation (it is best to have it move from right to left) must coincide with the zero line of the scale. This is accomplished with the help of a variable shunt (B in fig. 4) across the meter. The adjustment is only

valid of course for a given value of the light flux I_0, and this must be kept constant during the measurement (see below).

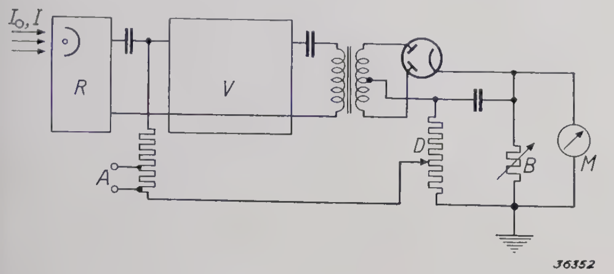

Fig. 4. Connections of the density meter. R photocell, V two-stage amplifier containing a variable mu pentode EF 9 and an output pentode. M voltmeter indicating directly the density, with shunt B. D potentiometer for regulating the fraction of the output voltage fed back to the valve EF 9. A calibration voltage can be applied at A.

The method of obtaining a linear density scale

If an ordinary amplifier with a linear amplification were used, the meter deviation would be proportional to I, *i.e.* we would obtain a logarithmic scale for the density: 90 per cent of the available scale length would be occupied by the region $Z = 0$ to $Z = 1$, the following 9 per cent by the region $Z = 1$ to $Z = 2$, and the last one per cent by the values of Z greater than 2. This is very undesirable, not only for radiation measurements, where the same degree of accuracy throughout the whole measuring region is desired, but also for judging the quality of the picture, because the impressions produced on the eye, according to the law of Weber and Fecher, are approximately proportional to the density.

In order to obtain a density scale which is approximately linear in the region of densities which is of practical importance, an amplifier would be needed which amplifies less with a large input signal (I) than with a small input signal. This requirement recalls the automatic volume control which is employed in radio receiving sets, and indeed the problem can here be solved in an entirely analogous manner [4]). As first amplifier valve we use a variable mu pentode EF 9; the negative grid bias of this valve is influenced by the rectified output voltage of the amplifier, so that the operating point on the valve characteristic is shifted to sections with steeper slope, as the input signal becomes smaller. The "control" characteristic, *i.e.* the relation obtained between output and input signal, still depends upon the fraction of the output voltage (adjustable with potentiometer D in fig. 4) fed back to the input, but in the case of the valve EF 9 it always has an approximately logarithmic section, with which a linear density scale can be obtained in the desired region (*fig.* 5).

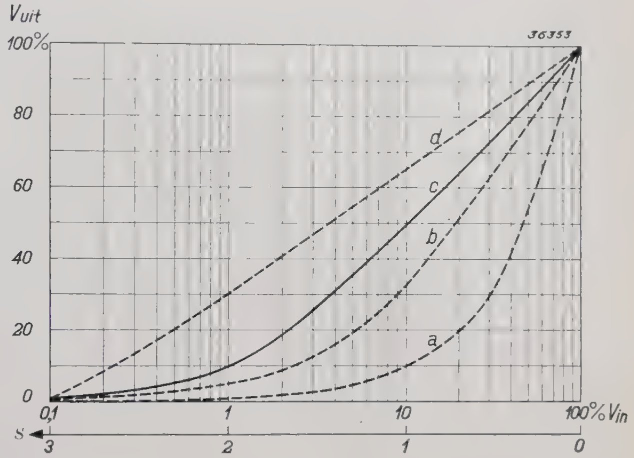

Fig. 5. Control characteristic (output voltage as a function of the input voltage, both in per cent of the value at $I = I_0$) for different adjustments of the potentiometer D in fig. 4.
a) No control; linear characteristic, very unfavourable scale.
b) Weak control (loss in amplification, a factor 4); for $0 < Z < 1$ the characteristic is already approximately logarithmic.
c) Control as used by us (loss in amplification, a factor 20); the characteristic is about logarithmic for $0 < Z < 2$.
d) Very strong control. While the density scale hereby obtained is linear up to $Z = 3$, the loss in amplification becomes undesirably great.

Calibration of the scale

By adjustment of the potentiometer D mentioned, the form of the control characteristic can be so influenced that the desired measuring region falls in a favourable position. Since in the case of X-ray films the region between $Z = 0$ and $Z = 2$ is of special importance, it was desirable in the case of the apparatus in question that the point $Z = 1$ should stand in the middle of the scale. This was accomplished in a very simple way. The signal caused by I_0 at the input of the amplifier was replaced by a calibration voltage (applied at A in fig. 4) which causes the same meter indication ($Z = 0$). This voltage was then lowered by a factor 10. Since the current of the photocell is exactly proportional to the light flux at the illumination intensities here prevailing, the attenuation of the calibration voltage by a factor 10 is exactly equivalent to the attenuation of the light flux by a film with the density $Z = 1$. By adjustment of D it was now brought about that the pointer of the measuring instrument coincided with the desired scale line in the middle of the scale (fig. 5 full line curve). In a similar way, namely by successively lowering the calibration voltage by known factors, the rest of the scale was then calibrated. In *fig.* 6 it may be seen that the scale is fairly approximately linear

[4]) See C. W. Miller, A linear photoelectric densitometer, Rev. sci. instrum. **6**, 125, 1935.

up to about $Z = 2$. The calibration was controlled by attenuating the light falling upon the photocell by known amounts, by changing the distance of the light source. The "density" measured always showed excellent agreement with the attenuation calculated from the inverse square law.

Fig. 6. Density scale of the apparatus with a linear scale below. It may be seen that the density scale is approximately linear in the region between $Z = 0$ and $Z = 2$, which is the most important for X-ray films. The ends $Z = 0$ and $Z = \infty$ of the scale must be checked before the beginning of every measurement and readjusted if necessary. The mechanical resting position of the pointer is on the extreme right, slightly beyond the point $Z = \infty$.

Several details of the construction

In *fig. 7* a photograph is given of the complete apparatus. In the horizontal box is the sodium lamp with the accompanying supply transformer. The supply voltage is stabilized. Since the light flux of the sodium lamp varies only very little with variations in the supply voltage, the light flux I_0 is practically entirely independent of mains voltage fluctuations. The lamp is cooled by a slight aïr current, since otherwise the outer bulb might become too hot.

The lamp illuminates a frosted glass plate (see fig. 3) which in turn serves as secondary light source for the diffuse illumination of the film being examined. The light passes through a diaphragm in the smoothly-finished upper surface of the box into a vertical tube blackened on the inside (fig. 7), and falls upon the photocell used as receiver. When the film is slid over the smooth plate so that the light falls upon the desired spot, the lower section of the tube is pulled down until it touches the film, so that the path of the light ray is entirely shielded from stray light. The diaphragm in the plate has a diameter of 5 mm and therefore the average density of a circle of that diameter is measured. If desired a smaller diaphragm can also be used.

The amplifier and the accompanying supply apparatus are housed in the vertical box. The supply voltage for the amplifier and the photocell is also stabilized, so that the calibration of the density scale is quite independent of fluctuations of the mains voltage.

The operation of the apparatus is limited to the control and if necessary to the readjustment of the two extremities of the scale before the beginning of each measurement. For setting the point $Z = 0$ with the shunt across the measuring instrument (B in fig. 4, the corresponding knob may be seen above on the left in fig. 7); one must of course wait until the light flux of the sodium lamp has reached its final value, which is the case about 15 minutes after switching on. The potentiometer D for choosing the correct control characteristic is not accessible from the outside; its adjustment needs only seldom to be verified, namely when the variable mu valve is exchanged.

Fig. 7. Photograph of the apparatus. The sodium lamp is in the horizontal box, on the smooth upper surface of this box the film being examined is placed. The light shines through the film into the vertical tube at whose upper end in the horizontal tube the photocell is mounted. The amplifier and the accompanying supply apparatus are housed in the upright box.

REAR LIGHTS AND REFLECTORS FOR BICYCLES

by H. A. E. KEITZ. 683.848.8 : 683.852 : 629.113.8

To protect the cyclist against the danger of being run into from behind by motorvehicles, rear lights or reflectors may be used. In certain countries fixtures are prescribed which function both as rear light and reflector. In this article the principles are discussed upon which the construction of these three types of safety devices is usually based. The Philips bicycle rear lamps, which are of the last-mentioned type, are dealt with in conclusion. The special measures are discussed which have been taken in order to determine easily whether or not the rear light is burning.

In recent years it has become more and more evident that cyclists must be protected at night against the danger of being run into from behind by motorist, and the use of rear lights and reflectors is being made compulsory in more and more countries. It is therefore of interest to examine the optical principles upon which the modern rear lights and reflectors for bicyles are based. Beginning with these optical principles we shall then discuss the way in which a rear lamp can at the same time be made to serve as a reflector, as is now prescribed in various countries. Finally a description will be given of the Philips bicycle rear lamps which belong to this type.

Rear lights

In the case of lamps which are intended only as rear lights, little need be said. Such lamps consist simply of a case with a window of red glass behind which a small lamp burns. This rear light need have an intensity of only a few millicandle power. In general lamps are used which are connected in parallel with the head lamp of the bicycle, and which consume a current of about 0.5 A with the normal running voltage of the bicycle dynamo. Since the current of a modern bicycle dynamo varies only little with the resistance of the lamps connected to it, when a rear light is required it is best to choose for the head lamp a lamp with a correspondingly lower nominal current, for instance 0.45 A instead of 0.5 A. A loss of light of 10 per cent is then experienced which will hardly be noticed, while with a lamp of nominally 0.5 A a loss of about 30 per cent would result, since the lamp would burn at a considerably lower temperature than with normal consumption.

The light intensity of the rear lamp is usually adequate with the simple arrangement mentioned [1]). If, however, an increase in the light intensity toward the rear is desired, this can be obtained at the expense of the acceptance angle of the beam by using a lens instead of the cover glass, and placing the lamp at a suitable spot between the lens surface and the focus. The closer the lamp is placed to the focus, the narrower the beam becomes and the greater the light intensity toward the rear.

Reflectors

Since the cyclist who may be considered to be in danger is practically always in the light beam emitted by the head lamps of the car overtaking him, it was at first thought sufficient if instead of a rear light a reflector was used which reflects the light incident from the head lamp to the car-driver's eye.

The rays falling upon the reflector may be considered as a parallel beam which must therefore be reflected as a practically parallel beam, and in the direction from which the light originally comes. A slight spreading about this direction is permissible and even necessary, since the direction from the reflector to the eye of the car driver does not exactly coincide with the direction from the reflector to the head lamp. The legal regulations existing in different countries about bicycle reflectors take this into account not only by specifying the intensity of the reflection in the direction of incidence, but also by requiring a definite reflective capacity within a definite solid angle around the direction of the incident light. This reflective capacity is usually given as the number of millicandle power reflected per lux of the incident light.

For the solution of the problem of reflecting an incident light beam in the direction of incidence there are two main principles which have led to practically useful constructions:

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[1]) According to the legal regulations of different countries the opinions about the necessary light intensity of a rear light are very divergent. A value of several millicandle power may in general be regarded as sufficient. The power of the ordinary rear lamps amounts to about 250 mW, which would in itself be more than adequate for the required light intensity. It must, however, be kept in mind that the cover glass may have quite a strong absorption, while, moreover, from considerations of safety margins, it is desirable to allow the rear light to burn at a relatively low temperature, so that the specific light yield is also quite low.

1) the triple mirror,
2) a lens with a mirror at the focus.

The triple mirror

In 1887 it was demonstrated by Beck [2]) that a system of three mutually perpendicular plane mirrors possesses the property that every light ray which is incident upon it is exactly reversed in direction, so that, apart from a lateral displacement whose maximum value is equal to the diameter of the opening between the three mirrors, it returns exactly to the point from which it was emitted (*fig. 1*).

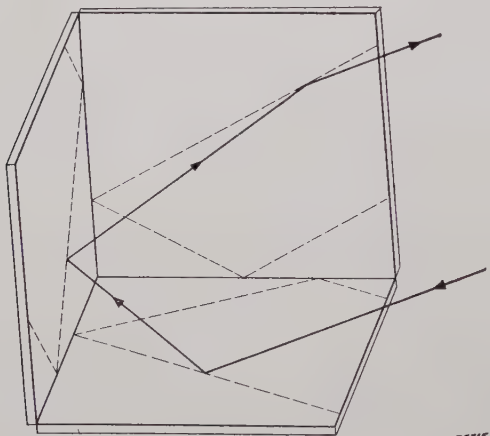

Fig. 1. Triple mirror. A ray, which is reflected by three mutually perpendicular planes, emerges from the third plane in a direction exactly opposite to the direction of incidence on the first plane.

This property of the triple mirror can be demonstrated quite simply by a slightly different interpretation of the law of reflection, which states that the angle of reflection is equal to the angle of incidence. For this purpose we resolve the direction vector of the incident rays into three mutually perpendicular components, two of which lie in the surface of the mirror, while the third is perpendicular to it (see *fig. 2*). It is now evident that upon reflection the component of the ray vector which is perpendicular to the mirror is reversed in direction, while the components parallel to the mirror remain unaltered. A glance at the figure shows immediately that in this case the angle of reflection is equal to the angle of incidence, while the incident ray, reflected ray and normal to the surface lie in a common plane as required by the law of reflection.

If we now pass over to the triple mirror we can best choose for the three components of the ray direction the normals to the three mirrors. Each normal is parallel to two other mirrors, so that we are again concerned with the case discussed above.

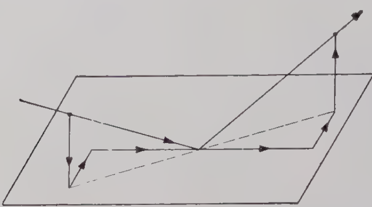

Fig. 2. Reflection on a plane mirror. The direction vector of the reflected ray may be found by reversing the component of the incident ray which is perpendicular to the reflecting surface, and leaving the components parallel to that surface unchanged.

If a ray is successively reflected by the three mirrors, one component of the ray vector is reversed in direction by each mirror, while the other two components remain unchanged. Finally all three components are reversed in direction, which means that the ray itself has been reversed in direction.

In the practical construction of a reflector on the principle of the triple mirror use may be made of the reflection experienced by a ray incident upon a glass surface. If a ray of light is allowed to fall perpendicularly upon the basal plane of a pyramid which has the shape of an obliquely cut off corner of a cube (see *fig. 3*), the ray is totally reflected at the three side surfaces of the pyramid (boundary surfaces glass-air) and then emerges again from the basal plane in a normal direction. The angle of incidence on the three side surfaces in this case is 54.7°. Since toatal reflection already takes place at considerably smaller angles, it is obvious that some-

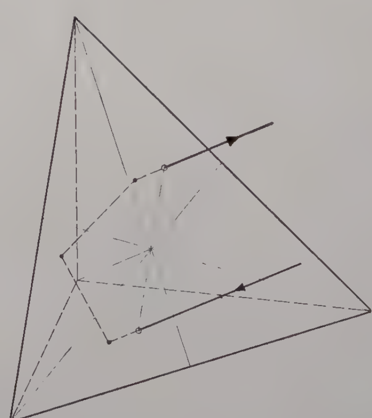

Fig. 3. A pyramid with three mutually perpendicular surfaces has the action of a triple mirror. The direction vector of a ray incident upon the basal plane of the pyramid is resolved into three components parallel to the three perpendiculars to the surfaces of the pyramid. Each of these components is perpendicular to one mirror surface and is reversed in direction when the ray is reflected by the surface in question.

[2]) A. Beck: Über einige neue Anwendungen ebener Spiegel, Z. Instr., **7**, 380-389, 1887.

what obliquely incident rays will also be totally reflected. For rays which have an angle of incidence on the basal plane of the pyramid of more than about 20°, the reflection is no longer total, but still very considerable; the reflectivity therefore decreases only slightly with increasing angle of incidence.

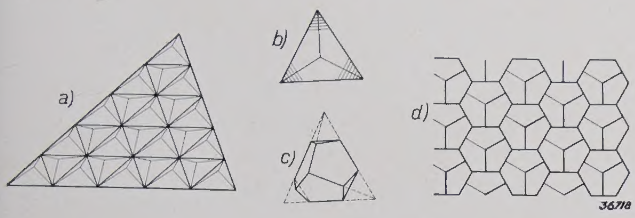

Fig. 4. Pyramid glass. *a*) Plane, covered with rectangular pyramids. A ray which is incident from below vertically upon one of these pyramids will also be reflected vertically downward, except when it is too close to one of the basal angles of the pyramid, namely in the shaded part of *b*). By cutting off this part of the pyramid as in *c*) the surface can be more completely covered with pyramids, for instance as in *d*), and in this way the efficiency of the reflector in the axis direction is increased.

The glass of the pyramid is red in order to give the reflected light the customary red warning colour. Since the path of the light through the glass is quite long, and since the loss of light must be limited in order to obtain the greatest possible efficiency, the coloration of the glass would have to be made extremely light with this type of construction. Such a weak coloration is difficult to obtain in mass production with sufficient constancy. Therefore in the construction of bicycle reflectors on this principle it is almost universally customary to limit the path of the light through the glass by impressing in the glass a large number of small pyramids side by side as indicated in *fig. 4a*, instead of using a single large pyramid. The part of the surface which is shaded in fig. 4*b* contributes no useful effect upon illumination in a perpendicular direction. If one considers how the reflected rays travel it is found that a ray which falls upon the shaded portion of the pyramid (the "dead angles") after two reflections emerges in an oblique direction without touching the third side. By cutting off the dead angles vertically according to fig. 4*c* a closer packing of pyramids can be obtained (see fig. 4*d*), and the efficiency of the reflector for perpendicularly incident rays is improved.

If the pyramid glass acted exactly as a triple mirror, the light of a head lamp would be reflected back to the head lamp in such a narrow beam that it would not meet the eye of the driver. It is therefore desirable that the surfaces of the pyramids should not reflect perfectly, but should have a certain spreading. This spreading is found in practice always to be present, because even the best constructed reflectors have deviations from the ideal form, particularly in the smoothness of the side surfaces of the pyramids. It is therefore no problem to obtain this spreading. On the contrary, great care must be taken to keep it small, since every increase in the spreading is accompanied by a decrease in the light intensity in the desired direction and therefore in the quality of the reflector.

In addition to careful finishing of the surfaces of the pyramids, a correct choice of the colour of the glass is of great importance for the quality of the reflector. Insufficient reflection may of course also be due to too dark a colour of the glass. It is therefore advantageous to choose the colour of the glass as light as is compatible with the requirement that the reflected light must be unequivocally observed as red.

Lens with a mirror at the focus

A lens with a mirror at its focus, like a triple mirror, has the property of reflecting every light ray which falls upon it in the direction from which it came (see *fig. 5*). On this basis bicycle reflectors can also be constructed which, although theoretically imperfect, since every lens possesses a spherical aberration, are practically just as satisfactory as reflectors which use a pyramid glass. As to the practical structure of a lens with a mirror, a distinction may be made between two types: one in which the rear surface of the lens forms the focal plane and is silvered so that it acts as a mirror, and one in which lens and mirror are separate. Several models of these two types may be seen in *fig. 6*.

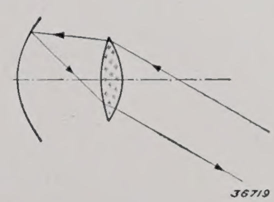

Fig. 5. Combination of lens and mirror. A ray which is reflected by a mirror at the focus of a lens emerges from the lens in a direction opposite to that of incidence.

With the large acceptance angle of the lenses, which is necessary to make possible a compact form of the reflectors, lenses with spherical surfaces cannot be used without obtaining non-permissible losses in the intensity of the reflected light, due to the very incomplete convergence of the rays at the focal plane. Since, however, the lens of the bicycle reflector is not ground, but pressed, the use of a non-spherical surface offers no difficulty in manufacture. A plane-convex lens is usually used, with the

convex side outermost. In order to be able to obtain an exact focussing in a single point, at least of the rays parallel to the axis, the curved surface of the lens must have approximately the form of an ellipsoid.

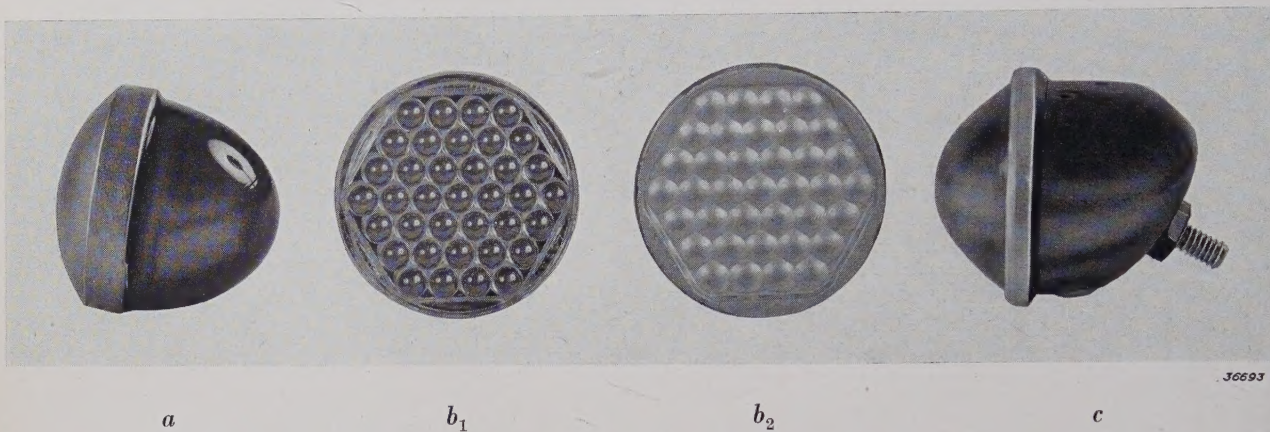

Fig. 6. Bicycle reflectors with combination of lens and mirror. In the types a, b_1 and b_2 the focal plane of the lens coincides with its rear surface; by covering the latter with a reflecting layer the desired effect is achieved (b_1 and b_2 are the front and back of the same reflector). In the type c) a separate mirror is used.

For obliquely incident light the focussing of the rays by the ellipsoid is not complete, so that the reflected beam is spread more and more with increasing angle of incidence. Since, however, for obliquely incident rays, less rigid requirements need be made than for rays parallel to the axis, this property of the lens-mirror combination is not objectionable.

Combination of rear light and reflector

The use of a reflector will in general furnish less perfect protection than the use of a rear light, but it has certain advantages; in the first place a reflector acts even when the bicycle is stationary, in the second place it is absolutely sure, while a rear light fails to act when the lamp or the connection between the lamp and the dynamo is defective. This objection to the rear light is particularly conclusive, since such a defect can scarcely be observed directly by the cyclist himself, and often escapes his attention for some time.

In order to meet this objection it is required in certain countries that a rear light must continue to act as a reflector when the lamp is extinguished. In addition methods have been sought in recent times of making the non-functioning of the rear light directly visible to the cyclist. These two possibilities will be considered further in the following.

As to the combination of rear light and reflector, different solutions are arrived at according as a reflector of the pyramid glass type or of the lens-mirror combination type is used. In the case of the first type it might be considered that it would be sufficient simply to place the lamp behind a covering of pyramid glass. This is found, however, not to constitute a possible solution of itself, since the pyramid glass is entirely opaque, at least in the direction perpendicular to the basal plane. The truth of this fact can easily be understood when it is kept in mind that light incident perpendicular to the front surface of the pyramids cannot pass through the glass but is totally reflected. From the reversibility of the light rays it follows directly that no light from a lamp behind the front surface can emerge in a direction perpendicular to it [3]).

If it is, nevertheless, desired to use a pyramid glass as cover of a rear light, care must be taken that the whole surface is not covered with pyramids. In *fig.* 7 two types of such cover glasses are shown: in the case of the first small windows are left open between the pyramids at the spots where the dead

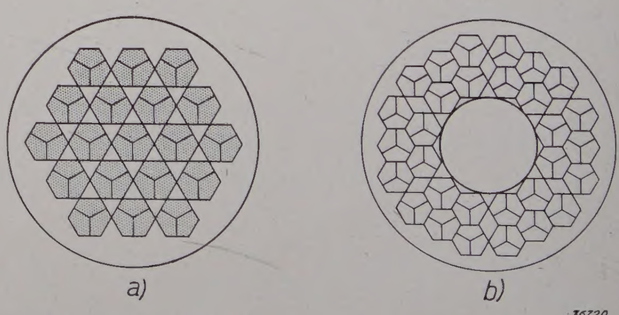

Fig. 7. Two types of pyramid glasses which may be used for rear lights.

[3]) This property may be used to pick out the poorly reflecting glasses. When a pyramid glass is held in front of a source of light the pyramids should remain dark. If, however, the side surfaces of the pyramids are very uneven they may allow light to pass through, and when this is the case it is a proof that the glass has poor reflective qualities.

angles of the pyramids would occur, in the case of the second a lens occupies the centre of the field and is surrounded by pyramids. The second type has been chosen for the rear light type 7/312 constructed by Philips and shown in *fig. 8*.

Fig. 8. The Philips bicycle rear light type 7 312 with pyramid glass.

If we now consider the possibilities which are offered by the combination of lens and mirror for the combining of reflector and rear light, we must make a distinction between the two forms of construction according to fig. 6*b* and 6*c*. The construction according to fig. 6*b* can be used by placing a large lens in the centre, instead of the small mirrored lenses, and behind it the lamp. If one wishes to use the combination of a lens and a mirror separated from each other (fig. 6*c*), the lamp can simply be placed between the lens and the mirror. Care must be taken that the path of the rays between lens and mirror is not too much interfered with by the presence of the lamp. A practical construction on this principle, the Philips rear light type 7 309, is shown in the cross section sketch *fig. 9*.

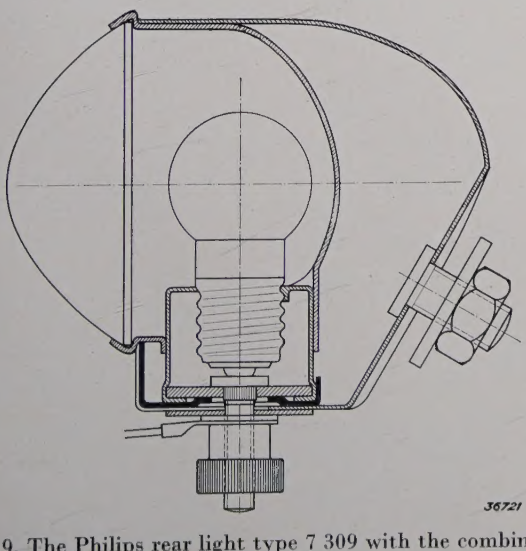

Fig. 9. The Philips rear light type 7 309 with the combination of a lens and a mirror.

Control of the rear light

As already mentioned, attempts have been made recently to find some method of making it directly evident to the cyclist whether or not the rear lamp is burning. The simplest solution is to construct the rear light so that by turning his head the cyclist can see for himself.

In order to be able to ascertain without stopping whether the rear light is burning, it is necessary that the lamp should emit a small amount of light in a forward direction obliquely upwards. In the case of the construction given in fig. 9 with a convex lens, this could be done simply by providing that the lens does not lie too deeply in the holder. By internal reflection and scattering a sufficient amount of light is then refracted in the desired direction. In constructions with a plane cover glass, for instance a pyramid glass, no lighted parts usually project beyond the holder. In the construction of

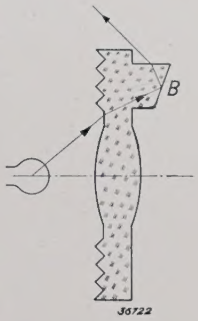

Fig. 10. Path of the rays in the prismatic projection of the Philips rear light type 7 312. The ray is totally reflected at the oblique front surface *B* of the prismatic projection and emerges from the glass in a direction such that it can be seen by the cyclist by turning his head.

the Philips rear light type 7 312 (fig. 8) a special solution was therefore devised of making the rear light visible to the cyclist without dismounting. This was accomplished by means of a prismatic projection at the upper edge of the glass, which can easily be seen by the cyclist as demonstrated in *fig. 10* where the path of the rays in this projection is sketched. It is obvious that the device described has no effect when the rear light is shielded from above, as is at present the case in many countries in connection with the black-out for air-raid defense.

Aside from this objection, the turning about of the cyclist also has other disadvantages connected with safety in traffic. Therefore in recent times another method has been adopted, namely that of connecting in series with the rear light a second lamp which is so placed that it may be seen without turning, and which indicates by remaining lighted that the circuit to the rear light is unbroken. In the case of certain models of Philips bicycle

lamps there is a special fitting for this control lamp in the back of the head lamp (see *fig. 11*). The control lamp is of the same type as the rear lamp; its burning temperature is, however, lower, since a resistance is connected in parallel with it.

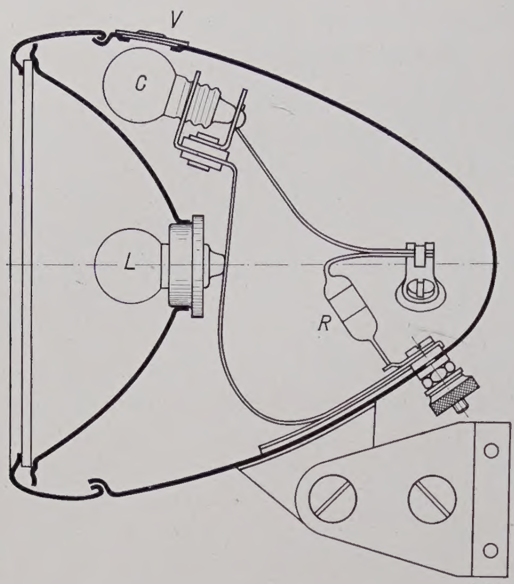

Fig. 11. Cross section of a Philips bicycle lamp with built-in control lamp for the rear light. *L* head lamp, *C* control lamp, *V* window, *R* resistance in parallel with the control lamp.

This construction has various favourable properties. In the first place it is convenient to have the same type of lamp for control lamp and rear light, it is consequently unnecessary to have two different types of spare lamps. In the second place the control lamp is only very lightly loaded due to the resistance in parallel with it, so that it practically never burns out [4]). If, however, due to some other cause, such as heavy shocks, the control lamp becomes defective, the circuit remains closed thanks to the resistance, so that the rear light continues to burn. If on the other hand the rear light becomes defective it can immediately be replaced by the control lamp; in this respect also therefore the control lamp contributes to the reliability of the rear light.

[4]) The fact that the light intensity of the control lamp is thus made very small is no disadvantage, but even an advantage, since it prevents blinding of the cyclist on dark roads.